1 What's in this document

- It gives in Section 2 the description of model B the way it will appear in
  - Den Haan, Wouter J., Kenneth L. Judd, and Michel Juillard, 2008, Computational suite of models with heterogeneous agents: Model Specifications, Journal of Economic Dynamics and Control this issue.
- It gives in Section 3 the description of the simulation procedure the way it will appear in
  - Den Haan, Wouter J., 2008, Comparison of solutions to the incomplete markets model with aggregate uncertainty, Journal of Economic Dynamics and Control, this issue.

2 Model B

This section gives the description of model B. Make sure your notation is exactly the same as the one used in this document. There have been slight changes in notation relative to the earlier draft.

- The wage rate is now simply w_t instead of $\overline{w}_t$.
- We use i to indicate the individual and j to use grid points. So make sure not to use, for example, i for grid points.
- We use ε instead of w to indicate employment status (in sub and superscripts). This is a bit inconsistent because in the budget constraint ε can be either 0 or 1 and as a superscript it can be either e or u but this shouldn't create any difficulties.

The economy is a production economy with aggregate shocks in which agents face different employment histories and partially insure themselves through (dis)saving in capital. For more details see Krusell and Smith (1998).

Problem for the individual agent. The economy consists of a unit mass of ex ante identical households. Each period, agents face an idiosyncratic shock ε that determines whether they are employed, $\varepsilon = 1$, or unemployed, $\varepsilon = 0$. An employed agent earns a wage rate of w_t and an after-tax wage rate of $(1-\tau_t)w_t$. An unemployed agent receives unemployment benefits μw_t. Note that Krusell and Smith set μ equal to zero. This is the only difference with their model. Markets are incomplete and the only investment available is capital accumulation. The net rate of return on this investment is equal to $r_t - \delta$, where r_t is the rental rate and δ is the depreciation rate. Agent's i maximization problem is as follows:

$$\max_{\{c_t^i, k_{t+1}^i\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^i\right)^{1-\gamma} - 1}{1-\gamma} \tag{1}$$

$$\text{s.t.} \quad c_t^i + k_{t+1}^i = r_t k_t^i + ((1-\tau_t)\bar{l}\varepsilon_t^i + \mu(1-\varepsilon_t^i))w_t + (1-\delta)k_t^i \tag{2}$$

$$k_{t+1}^i \geq 0 \tag{3}$$

Here c_t^i is the individual level of consumption, k_t^i is the agent's beginning-of-period capital, and $\bar{l}$ is the time endowment.

Firm problem. Markets are competitive and the production technology of the firm is characterized by a Cobb-Douglas production function. Consequently, firm heterogeneity is not an issue. Let K_t and L_t stand for per capita capital and the employment rate, respectively. Per capita output is given by

$$Y_t = a_t K_t^{\alpha} (\bar{l} L_t)^{1-\alpha} \tag{4}$$

and prices by

$$w_t = (1-\alpha)\, a_t (\frac{K_t}{\bar{l} L_t})^{\alpha} \tag{5}$$

$$r_t = \alpha a_t (\frac{K_t}{\bar{l} L_t})^{\alpha - 1} \tag{6}$$

Aggregate productivity, a_t, is an exogenous stochastic process that can take on two values, $1 - \Delta^a$ and $1 + \Delta^a$.

Government The only role of the government is to tax employed agents and to redistribute funds to the unemployed. We assume that the government's budget is balanced each period. This implies that the tax rate is equal to

$$\tau_t = \frac{\mu u_t}{\bar{l} L_t}. \tag{7}$$

where $u_t = 1 - L_t$ denotes the unemployment rate in period t.

Exogenous driving processes. There are two stochastic driving processes. The first is aggregate productivity and the second is the employment status. Both are assumed to be first-order Markov processes. We let $\pi_{aa'\varepsilon\varepsilon'}$ stand for the probability that $a_{t+1} = a'$ and $\varepsilon^i_{t+1} = \varepsilon'$ when $a_t = a'$ and $\varepsilon^i_t = \varepsilon'$. These transition probabilities are chosen such that the unemployment rate is a function of a only and can, thus, take on only two values. That is, $u_t = u(a_t)$ with $u^b = u(1-\Delta^a) > u^g = u(1+\Delta^a)$.

Parameter values Two sets of parameter values are considered. In the first economy, there is no aggregate uncertainty. The transition probabilities for the idiosyncratic shock correspond to those of Krusell and Smith (1998) when the economy is always in the bad state. The aggregate capital stock is given and fixed, which results in a constant interest rate.

The parameter values of the second economy correspond with those of Krusell and Smith (1998) except that the unemployed receive unemployment benefits. Its parameter values are given in Tables 1 and 2. The discount rate, coefficient of relative risk aversion, share of capital in GDP, and the depreciation rate take on standard values. Unemployed people are assumed to earn a fixed fraction of 15% of the wage of the employed. The value of Δ^a is equal to 0.01 so that productivity in a boom, 1+Δ^a, is two percent above the value of productivity in a recession, 1-Δ^a. Business cycles are symmetric and the expected duration of staying in the same regime is eight quarters. The unemployment rate in a boom, u^g, is equal to 4% and the unemployment rate in a recession, u^b, is equal to 10%. The time endowment, $\bar{l}$, is chosen to normalize total labor supply in the recession to one. The average unemployment duration is 2.5 quarters conditional on staying in a recession and equal to 1.5 quarters conditional on staying in a boom. These features correspond with the transition probabilities reported in Table 2.

The parameter values of the economy without aggregate uncertainty are identical to those of the economy with aggregate uncertainty with the following exceptions. Δ^a is set equal to zero and the aggregate capital stock is held constant at 43. The unemployment rate is always equal to μ^b and the transition probabilities are given in Table 3.

Table 1: Benchmark calibration

Parameters	β	γ	α	δ	$\bar{l}$	μ	Δ^a
Values	0.99	1	0.36	0.025	1/0.9	0.15	0.01

Table 2: Transition probabilities

s,ε / s',ε'	1-$\Delta^a, 0$	1-$\Delta^a, 1$	1+$\Delta^a, 0$	1+$\Delta^a, 1$
1-$\Delta^a, 0$	0.525	0.35	0.03125	0.09375
1-$\Delta^a, 1$	0.038889	0.836111	0.002083	0.122917
1+$\Delta^a, 0$	0.09375	0.03125	0.291667	0.583333
1+$\Delta^a, 1$	0.009115	0.115885	0.024306	0.850694

Table 3: Transition probabilities model without aggregate uncertainty

ε / ε'	0	1
0	0.6	0.4
1	0.044445	0.955555

3 A non-stochastic simulation procedure

Information used. The beginning-of-period t distribution of capital holdings is fully characterized by the following:

- the fraction of unemployed agents with a zero capital stock, $p_t^{u,0}$,
- the fraction of employed agents with a zero capital stock,[1] $p_t^{e,0}$,
- the distribution of capital holdings of unemployed agents with positive capital holdings, and
- the distribution of capital holdings of employed agents with positive capital holdings.

Overview. The goal is to calculate the same information at the beginning of the next period. Besides these four pieces of information regarding the cross-sectional distribution one only needs (i) the realizations of the aggregate shock this period and next period and (ii) the individual policy function.

Grid Construct the following grid and define the begining-of-period distribution of capital as follows.

- $\kappa_0 = 0$ and $\kappa_j = 0.1j$, $j = 1, \cdots, 1000$.

[1] Employed agents never choose a zero capital stock but some unemployed agents that chose a zero capital stock last period have become employed this period.

- Let $p_t^{\varepsilon,0}$ be the fraction of agents with employment status ε with a zero capital stock at the beginning of period t.
- For $j > 0$, let $p_t^{\varepsilon,j}$ be equal to the mass of agents with a capital stock bigger than κ_{i-1} and less than or equal to κ_i. This mass is assumed to be distributed uniformly between gridpoints.
- We have

$$\sum_{j=0}^{1000} p_t^{u,j} = 1, \quad \sum_{j=0}^{1000} p_t^{e,j} = 1.$$

Denote this beginning-of-period distribution function by $P_t^{\varepsilon}(k)$.

The initial distribution is also made availabe at http://www1.fee.uva.nl/toe/content/people/content/denhaan/datasuite.htm.

End-of-period distribution The first step is to calculate the end-of-period distribution of capital.

For the unemployed calculate the level of capital holdings at which the agent chooses κ_j. If we denote this capital level by $x_t^{u,j}$ then it is defined by[2]

$$k'(x_t^{u,j}, \cdot) = \kappa_j. \tag{8}$$

Now compute the end-of-period distribution function at the grid points as

$$F_t^{u,j} = \int_0^{x_t^{u,j}} dP_t^u(k) = \sum_{j=0}^{\overline{j_u}} p_t^{u,j} + \frac{x_t^{u,j} - \kappa_{\overline{j_u}}}{\kappa_{1+\overline{j_u}} - \kappa_{\overline{j_u}}} p_t^{u,\overline{j_u}+1}, \tag{9}$$

where $\overline{j_u} = \overline{j}(x_t^{u,j})$ is the largest value of j such that $\kappa_j \leq x_t^{u,j}$. The second equality follows from the assumption that P_t^u is distributed uniformly between gridpoints.

A similar procedure is used to calculate the end-of-period distribution for the employed.

$$F_t^{e,j} = \int_0^{x_t^{e,j}} dP_t^e(k) = \sum_{j=0}^{\overline{j_e}} p_t^{e,j} + \frac{x_t^{e,j} - \kappa_{\overline{j_e}}}{\kappa_{1+\overline{j_e}} - \kappa_{\overline{j_e}}} p_t^{e,\overline{j_e}+1},$$

where $\overline{i_j} = \overline{i}(x_t^{e,j})$ is the largest value of j such that $\kappa_j \leq x_t^{e,j}$.

[2] This is a non-linear problem (and has to be calculated at many nodes) but it should be a well behaved problem.

Next period's beginning-of-period distribution Let $g_{\varepsilon_t \varepsilon_{t+1} a_t a_{t+1}}$ stand for the mass of agents with employment status ε that have employment status ε_{t+1}, conditional on the values of a_t and a_{t+1}. For each combination of values of a_t and a_{t+1} we have

$$g_{u_t u_{t+1} a_t a_{t+1}} + g_{e_t u_{t+1} a_t a_{t+1}} + g_{u_t e_{t+1} a_t a_{t+1}} + g_{e_t e_{t+1} a_t a_{t+1}} = 1. \tag{10}$$

We then have

$$P_{t+1}^{\varepsilon,j} = \frac{g_{u_t \varepsilon_{t+1}}}{g_{u_t \varepsilon_{t+1}} + g_{e_t \varepsilon_{t+1}}} F_t^{u,j} + \frac{g_{e_t \varepsilon_{t+1}}}{g_{u_t \varepsilon_{t+1}} + g_{e_t \varepsilon_{t+1}}} F_t^{e,j} \tag{11}$$

and

$$p_{t+1}^{\varepsilon,0} = P_{t+1}^{\varepsilon,0} \tag{12}$$

$$p_{t+1}^{\varepsilon,j} = P_{t+1}^{\varepsilon,j} - P_{t+1}^{\varepsilon,j-1} \tag{13}$$

Note that to implement this procedure and to ensure that differences in the simulated output is only due to differences in the policy functions used we have to use the same interval length, which is set equal to 0.1. Since setting the upperbound can be an important part of the program, participants are free to set their own upperbound.