The Role of Debt and Equity Finance over the Business Cycle

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Abstract

The standard framework of firm finance that is used in DSGE models to study the role of financial frictions for business cycles only allows for debt finance and describes firm behavior using a representative firm. However, equity finance is important and the observed cyclical behavior of firm finance is different for firms of different size. This paper analyzes the cyclical behavior of firm finance in a framework that uses the standard setup for debt finance, but adds the possibility for firms to raise equity. Allowing for equity issuance overturns two undesirable features of the standard debt contract: a procyclical default rate and dampening of shocks. The model can explain the size-dependent procyclical behavior of equity issuance that is observed for most listed firms, i.e., the set of firms that excludes the largest firms.

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1 Introduction

Frictions in obtaining firm finance are believed to be important for business cycles and there are numerous articles that build such frictions into DSGE models.[1] These models typically only allow for debt finance.[2] In reality, however, firms often issue equity and equity issues are quantitatively important.[3] The question arises how relevant such studies are when they leave out an important source of firm finance. For example, if firms can avoid a tightening of frictions in debt financing by replacing debt with equity finance, then models that only allow for debt financing could overstate the importance of financial frictions.

There are some models described in the literature in which firms do have access to both debt and equity issuance. Jermann and Quadrini (2006) build a theory in which debt issuance is procyclical and equity issuance is countercyclical. The reason for these results is that the constraint on debt financing is tightened during a recession, which leads to a substitution out of debt and into equity financing. In contrast, Levy and Hennessy (2007) build a theory in which it is the constraint on *equity* financing that tightens during a recession.

The empirical results also seem to contradict each other. Whereas Jermann and Quadrini (2006) find that debt is procyclical and equity issuance is countercyclical, Korajczyk and Levy (2003) find that equity issuance is procyclical and debt issuance is countercyclical. There are two reasons for these seemingly contradicting results. First, as discussed in Fama and French (2001, 2005), measuring net equity issuance is a complex and difficult task.[4] Second, Covas and Den Haan (2010a) document that the cyclical

[1] Examples of early papers are Bernanke and Gertler (1990), Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999), but many papers have followed suit.

[2] An alternative approach that has been pursued is based on the assumption that firms can obtain funds through an "optimal" contract. Although, such contracts may combine features of debt and equity contracts, it is questionable that models with optimal contracts accurately capture the essence of observed firm financing practices in which debt and equity contracts play an important and (to a large extent) distinct role.

[3] Fama and French (2005) document the importance of equity issuance.

[4] Covas and Den Haan (2010b) also discuss measurement issues and in particular the drawbacks of using

behavior of equity issuance is not the same for all firms, but depends strongly on firm size. As summarized in Section 2, equity issuance is robustly procyclical for most firms, definitely for firms in the bottom 90%. But strong countercyclical behavior is found for the top 1%.[5] Moreover, as documented in Covas and Den Haan (2010a), the largest firms are so large that their behavior is important for the aggregate time series.

It will probably not be an easy task to build a model that can explain this rich set of empirical findings. But given the importance of equity issuance and the believed importance of financial frictions for understanding business cycles, it is important to build theoretical models in which firms use both debt and equity financing.

In this paper, we make a modest first step. We start with the standard framework in which the only form of external finance is debt and there is an unavoidable deadweight loss if the firm's revenues are insufficient to cover the repayment of the debt principal and the agreed upon interest payment. We will refer to this friction as the bankruptcy friction. We modify this model by allowing firms to also raise equity. Regarding the friction that firms face in raising equity finance, we follow Cooley and Quadrini (2001) and assume that equity issuance costs are increasing with the amount of equity raised. In addition, we allow for this cost to depend on the business cycle. Our framework is very simple, except that there is a continuum of heterogeneous firms. This allows us to analyze the relationship between firm size and the cyclicality of firm finance, which is an important feature of the data.

The simplicity of the framework makes it possible to describe the interaction between the cyclical behavior of debt and equity issuance. It also reveals under which conditions one can expect the model with equity issuance to have different predictions than the standard model with only firm finance.

The key results of the paper are the following. First, the model presented in this paper, i.e., the standard framework modified to allow for equity issuance, predicts that equity issuance is procyclical. Thus, this model would be appropriate for many firms, but not for

the flow of funds data. It is also shown that the flow of funds equity data by leveraged buyouts, which are important mainly for the largest firms.

[5] The results are mixed for firms in between the 90^{th} and the 99^{th} percentile.

the largest firms. Second, equity issuance is more procyclical for the smaller firms, which is consistent with the data. However, the model does not generate countercyclical net equity issuance for the largest firms, which is also observed in the data. Third, the model with equity issuance overcomes two of the drawbacks of the standard model with only debt financing. First, if the friction to raise equity is sufficiently countercyclical, then the default rate is no longer procyclical. Second (and related to the first property), shocks are magnified in the model with both debt and equity financing, whereas shocks are *dampened* by the presence of frictions in the standard model with only debt financing.

The rest of this paper is organized as follows. Section 2 summarizes the main stylized facts regarding the cyclical properties of U.S. debt and equity finance for different size-sorted firm groups. In Section 3, we use a two-period version of our framework to generate analytic results regarding the cyclicality of debt and equity issuance. Section 4 presents the full dynamic model and discusses the calibration. Section 5 documents in which dimensions the model successfully describes the observed cyclicality of debt and equity issuance. Section 6 documents how equity issuance can overturn the procyclical default rate, which is a unwelcome feature of the standard debt contract, and can turn the dampening of the standard debt contract into magnification. The last section concludes.

2 Observed cyclicality of U.S. debt and equity issuance

In this section, we summarize the results found in Covas and Den Haan (2010a). In this paper, we focus mainly on the results for the bottom 95%. We start this section with the reasons for doing this.

Excluding the largest firms from the analysis. Although small in number, firms in the top 1% and the top 5% are so large that even observations of single firms have noticeable effects on the *aggregate* series. Moreover, the behavior of these largest firms, especially the top 1%, is quite different from that of the other firms. A notable example is the huge increase in equity issuance by AT&T Corp in the run up of its forced breakup in 1983, i.e., during an economic recession.

Whether the results for firms in the [90%,95%] group are like the results for the smaller or the larger firms depends on the particular definition of equity issuance. For most series considered in Covas and Den Haan (2010a), the cyclical behavior of equity issuance by firms in this group resembles qualitatively the cyclical behavior by firms in the bottom 90%, that is, in terms of being procyclical or countercyclical. But it must be pointed out that the results for this subgroup are not as robust as those for the bottom 90%. The situation is similar for firms in the [95%,99%] group. In terms of the qualitative features their statistics resemble those of the bottom 90% for several equity measures, but the results are even less robust than for the [90%,95%] firm group.

Although few in number, these largest firms are so large that they are important for the cyclical behavior of aggregate equity issuance. For example, for our preferred equity measure we find that the correlation of equity issuance and GDP is equal to 0.46 and significant for the bottom 95%. When we consider the bottom 99%, this number drops to 0.35, but it remains significant. When we just add the top 1%, then this number plummets to an insignificant 0.17.[6] The quantitative importance of the top 1% is quite remarkable given that the average number of firms in the top 1% is only 32.

The upshot is that in terms of qualitative results a uniform picture emerges if one focuses on the bottom 90% and possibly the bottom 95%. The results for the largest firms are either clearly different (the results for the top 1%) or not very robust.

Debt and equity measures considered. Covas and Den Haan (2010a) consider several measures for debt and equity issuance by US firms. In this paper, we focus on the *net* amount of equity raised and the *net* increase in total liabilities raised.[7] Our measures are comprehensive and include for example equity raised through options being issued and

[6] Covas and Den Haan (2010a) report that adding the top 1% changes the cyclicality of equity issuance from procyclical to countercyclical when new firms are excluded from the analysis, although the countercyclicality is not significant. Jermann and Quadrini (2006) also report countercyclical behavior for aggregate equity issuance.

[7] Covas and Den Haan (2010a,b) document that neither the choice of series nor the way firm groups or cyclical measure are constructed affect the conclusions as long as one avoids equity series that are known to have severe measurement problems.

trade credit. In a macroeconomic study that focuses on the interaction between the ability of firms to raise external funds and real activity, it is important to use measures that are broad and give a good estimate of *all* the funds raised by firms through debt and equity contracts. For the same reason, it is important to realize that we focus on the actual amount of funds being raised, not on changes in the market value of existing liabilities or net worth.

Summary statistics. Table 1 documents key information about the size-sorted firm groups used in the empirical analysis in Covas and Den Haan (2010a).[8] The following observations can be made. Smaller firms grow faster than larger firms. This is true in terms of employment and in terms of assets owned. For small firms, the largest share of the increase in assets is financed by equity. For example, for firms in the bottom quartile, this fraction is 88%. For larger firms, debt becomes more important. For example, for firms in the [75%,90%] group, 60% of asset growth is financed by an increase in liabilities. Retained earnings are also more important for large firms than for small firms.[9] Although there are on average only 32 (out of 3128) firms in the top 1%, they own roughly 33% of all assets.

Cyclicality of debt and equity issuance. The first column of Table 2 reports the correlations between GDP and the net change in total liabilities as well as the correlations between GDP and net equity issuance. The results are reported for the different firm groups. All measures are detrended using the HP filter.[10]

Correlation coefficients are less appropriate to document the magnitude of fluctuations over the business cycle. Table 3 reports the panel regression estimates of Covas and Den Haan (2010a) for the net increase in debt and equity as a fraction of (lagged) assets

[8] Book value of assets is used to form firm groups. See Covas and Den Haan (2010a) for exact definitions.

[9] For firms in the bottom quartile, retained earnings (and profits) are on average negative. This is to a large extent due to small firms making negative profits in the second half of the nineties (during the dotcom bubble). When a longer sample is considered, then internal finance is on average no longer negative for this firm group, but it remains unimportant as a source of financing firm growth.

[10] Throughout this paper we use a smoothing coefficient of 100 to filter the annual data.

when the cyclical indicator improves with one standard deviation.

The findings reported in the two tables can be summarized as follows: (i) debt issuance is significantly procyclical for all firm groups, (ii) equity issuance is significantly procyclical for all firm groups in the bottom 95%,[11] (iii) equity issuance is procyclical but not significant for firms in the [95%,99%] firm group, (iv) equity issuance is significantly *counter*cyclical for firms in the top 1%, and (v) quantitatively the cyclicality of equity issuance is stronger for smaller firms and monotonically declining for the firm groups in the bottom 90%. The results found for U.S. firms are confirmed for Canadian firms in Covas and Den Haan (2006).

3 Adding equity issuance to a standard model of debt finance

In this section, we use a two-period model that is simple enough to derive analytical results about the cyclical behavior of equity issuance. Despite the model's simplicity, the model contains the key ingredients needed to highlight the interaction between the friction in obtaining debt finance and the cyclicality of equity issuance.

In period 1, firms invest k units in capital and this is financed using internal funds and two forms of external finance, namely debt and equity. In period 2, the firms' revenues, that are subject to an idiosyncratic shock, ω, are realized. If the firm's revenues are not enough to pay back the debt providers, then the firm defaults and its revenues are reduced by a deadweight loss. This follows the standard setup underlying DSGE models that incorporate financial frictions, except that we allow for both debt and equity issuance instead of just debt.[12]

As discussed in the previous section, equity issuance is quantitatively too important to be ignored in models about firm finance. The question arises how to best model the frictions firms face in obtaining equity finance. We start out with an *acyclical* quadratic

[11] As pointed out above, the results are not that robust for the [90%,95%] firm group.

[12] Two exemplary papers that incorporate this framework of financial frictions in a DSGE model are Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999).

adjustment cost. By using a friction that is itself acyclical, we do not bias the results one way or the other. This setup helps us to understand the forces in the model that make equity issuance countercyclical or procyclical when the alternative form of external finance is debt finance.

3.1 Debt contract and bankruptcy friction

Technology is given by

$$\theta\omega k^{\alpha} + (1-\delta)k, \tag{1}$$

where k stands for the amount of capital, θ for the aggregate productivity shock (with $\theta > 0$), ω for the idiosyncratic productivity shock (with $\omega \geq 0$ and $\mathrm{E}(\omega) = 1$), and δ for the depreciation rate. The value of θ is known when the debt contract is written, but ω is only known when revenues are realized.

It is standard in the literature to assume that technology is linear, that is, $\alpha = 1$.[13] The linearity assumption is convenient for computational reasons, since it means that agency costs do not depend on firm size and a representative firm can be used. Neither the assumption itself nor the implication that firm size does not matter is appealing. Therefore, we use a standard nonlinear production function.

The firm's net worth is equal to n and debt finance occurs through one-period contracts. That is, the borrower and lender agree on a debt amount, $(k - n)$, and a borrowing rate, r_b. The firm defaults if resources in the firm are not enough to pay back the amount due. That is, the firm defaults if ω is less than the default threshold, $\overline{\omega}$, where $\overline{\omega}$ satisfies

$$\theta\overline{\omega}k^{\alpha} + (1-\delta)k = (1+r_b)(k-n). \tag{2}$$

If the firm defaults, then the lender gets

$$\theta\omega k^{\alpha} + (1-\delta)k - \mu\theta k^{\alpha}, \tag{3}$$

[13] An economy in which aggregate production is linear in capital is not very realistic. To deal with this dilemma one typically assumes that there is also another sector that does have diminishing returns, but that—to keep computational complexity low—faces no frictions in obtaining external funds. By using a nonlinear solution algorithm, we can analyze a model in which firms face both diminishing returns and financial frictions.

where μ represents bankruptcy costs, which are assumed to be a fraction of expected revenues. In an economy with $\mu > 0$, defaults are inefficient and they would not happen if the first-best solution could be implemented. Bankruptcy costs are assumed to be unavoidable, however, and the borrower and the lender cannot renegotiate the contract. The idea is that bankruptcy is like a distress state, involving, for example, loss of confidence, loss of sales, fire sales of assets, and loss of profits.[14]

Using Equation (2), the firm's expected income can be written as

$$\theta k^{\alpha} F(\overline{\omega}) \text{ with } F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega}))\overline{\omega}, \tag{4}$$

and the lender's expected revenues as

$$\theta k^{\alpha} G(\overline{\omega}) + (1-\delta)k \text{ with } G(\overline{\omega}) = 1 - F(\overline{\omega}) - \mu\Phi(\overline{\omega}), \tag{5}$$

where $\Phi(\omega)$ is the cumulative distribution function (CDF) of the idiosyncratic productivity shock, which we assume to be differentiable.

The values of k and $\overline{\omega}$ are chosen to maximize the expected end-of-period firm income subject to the constraint that the lender must at least break even. Thus,

$$\begin{aligned} w(n;\theta) = & \max_{k,\overline{\omega}} \ \theta k^{\alpha} F(\overline{\omega}) \\ \text{s.t. } & \theta k^{\alpha} G(\overline{\omega}) + (1-\delta)k - (1+r)(k-n) > 0 \end{aligned} \tag{6}$$

The lower the depreciation rate, the larger the share of available resources that is not subject to idiosyncratic risk and the more the firm can borrow.

[14] In the framework of Townsend (1979), bankruptcy costs are *verification* costs and debt is the optimal contract. It is not clear to us, however, that verification costs are large enough to induce quantitatively interesting agency problems. Indeed, Carlstrom and Fuerst (1997) include estimates for lost sales and lost profits, and assume that bankruptcy costs are 25% of the value of the capital stock in their calibration. Under this alternative interpretation of bankruptcy costs, one can no longer use optimality properties to motivate the use of debt. In our analysis, we simply take the existence of debt and equity contracts as given in our analysis. Convenience and past practice are likely to be important reasons behind the continued use of debt and equity contracts in obtaining external finance.

For an interior solution, the optimal values for k and $\overline{\omega}$ satisfy the break-even condition of the bank and the first-order condition:

$$\frac{\alpha\theta k^{\alpha-1}F(\overline{\omega})}{\delta + r - \alpha\theta k^{\alpha-1}G(\overline{\omega})} = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})}. \tag{7}$$

The Lagrange multiplier of the bank's break-even condition, ζ, can be expressed as a function of $\overline{\omega}$ alone, and is always greater or equal to one. That is,

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1 - \mu\Phi'(\overline{\omega})/(1 - \Phi(\overline{\omega}))} \geq 1. \tag{8}$$

We make the following assumption about the debt contract.

Assumption A.

- *The maximization problem has an interior solution.*[15]
- *At the optimal value of $\overline{\omega}$, the CDF satisfies*

$$\frac{\partial \left(\Phi'(\omega)/(1 - \Phi(\omega))\right)}{\partial \omega} > 0. \tag{9}$$

This inequality is a weak condition and is satisfied if the density, $\Phi'(\omega)$, is non-zero and non-decreasing at $\overline{\omega}$.[16]

3.2 Equity contract

We follow Cooley and Quadrini (2001) and assume that equity issuance costs are increasing with the amount of equity raised. Whereas Cooley and Quadrini (2001) assume that the cost of issuing equity is linear, we assume that these costs are quadratic;

[15] Although unlikely, the model could have a corner solution. For example, if aggregate productivity is low, depreciation is high, bankruptcy costs are high, and/or the CDF of ω has a lot of mass close to zero, then it may be optimal not to borrow at all.

[16] Such an assumption is standard in the literature. For example, Bernanke, Gertler, and Gilchrist (1999) assume that $\partial \left(\omega d\Phi(\omega)/(1 - \Phi(\omega))\right)/\partial \omega > 0$, which would be the corresponding condition if bankruptcy costs are—as in Bernanke, Gertler, and Gilchrist (1999)—a fraction of actual (as opposed to expected) revenues.

that is, $\lambda(e) = \lambda_0 e^2$ for $e \geq 0$.[17] One reason for equity issuance costs is the presence of underwriting fees.[18] Alternatively, one could interpret the equity issuance costs as a reduced-form representation for losses due to an adverse-selection problem that firms face when convincing others to become co-owners. The question arises as to whether such an adverse-selection problem should not be modelled jointly with the debt problem.[19] To some extent it probably should, and it would be worthwhile to construct a framework in which the agency problem firms face in raising one type of external finance interacts directly with the agency problem firms face in raising the other type of external finance. We limit ourselves, however, to the question how the standard debt contract affects the cyclicality of equity issuance when there is no such interaction between the two frictions.

The firm chooses the amount of debt and equity financing simultaneously, together with the level of investment. To simplify the exposition, we describe the problem as if the problem is sequential. That is, a firm with internal resources equal to x, first chooses the level of equity financing, e. Next, the firm solves the debt problem described above with its net worth level given by $n = x + e$.

Recall that $w(n;\theta)$ is the expected end-of-period value of a firm that starts with net worth equal to n. The equity issuance decision is represented by the following maximization problem:

$$\begin{aligned} v(x;\theta) &= \max_{e,s} \tfrac{(1-s)w(x+e;\theta)}{1+r} \\ \text{s.t.} \quad e &= \tfrac{s\,w(x+e;\theta)}{1+r} - \lambda(e), \end{aligned} \tag{10}$$

where s is the ownership fraction that the providers of new equity obtain in exchange for e. In this specification, it is assumed that the equity issuance costs are paid by the outside investor, but this is irrelevant.[20]

[17] This avoids a non-differentiability when zero equity is being issued. Jermann and Quadrini (2006) also assume a quadratic cost of issuing equity.

[18] In fact, Hansen and Torregrosa (1992) and Altinkiliç and Hansen (2000) show that underwriting fees display increasing marginal costs.

[19] For example, if the equity provider also provides debt, then it may be easier to renegotiate the debt and avoid bankruptcy costs.

[20] Both the maximization problem in (10) and the problem in which issuance costs are paid by the firm correspond to maximizing $w(x + e;\theta)/(1 + r) - e - \lambda(e)$ with respect to e.

The expected rate of return for equity providers is equal to

$$\frac{sw(x+e,\theta)-(e+\lambda(e))}{e+\lambda(e)}=\frac{(1+r)\,(e+\lambda(e))-(e+\lambda(e))}{e+\lambda(e)}=r. \tag{11}$$

That is, providers of equity financing obtain the same expected rate of return as debt providers.

The first-order condition for the equity issuance problem is given by

$$\frac{1}{1+r}\frac{\partial w(x+e;\theta)}{\partial e}=1+\frac{\partial\lambda(e)}{\partial e}. \tag{12}$$

That is, the marginal cost of issuing one unit of equity, $1+\partial\lambda/\partial e$, has to equal the expected benefit. Since $\partial\lambda/\partial e$ is equal to zero at $e=0$, the firm will issue equity whenever $\partial w/\partial e>1+r$. The firm does not increase equity up to the point where $\partial w/\partial e=1+r$, since $\partial\lambda/\partial e>0$ for $e>0$.

3.3 Standard debt contract and the cyclicality of equity

In this section, we address the question how equity issuance responds to an increase in aggregate productivity when the friction affecting equity issuance is itself acyclical and debt is the only other form of external finance. The value of available equity finance clearly depends on the ease with which debt can be acquired. Using the envelope condition, we get directly from the debt problem that

$$\frac{\partial w(x+e;\theta)}{\partial e}=\zeta, \tag{13}$$

that is, the marginal value to the firm of having an extra unit of equity is equal to the Lagrange multiplier on the bank break-even constraint. Thus, the value of an extra unit of equity increases as frictions in obtaining debt financing intensify and ζ rises. Our specification of equity issuance costs—and thus the specification for the marginal cost of issuing equity—is by construction acyclical, that is, $\partial\lambda\left(e\right)/\partial e$ does not depend on θ. Since the marginal cost curve, $1+\partial\lambda(e)/\partial e$, is fixed and the marginal revenue curve, $\partial w(x+e)/\partial e$, shifts up with ζ, it follows immediately that equity issuance should increase when ζ increases, that is, when the friction of obtaining debt increases.

If the shadow price of debt, ζ, increases during a recession, then our model would have the prediction that equity issuance would increase during a recession, that is, equity issuance would be *counter*cyclical. The interesting feature of the standard debt contract, however, is that ζ is itself countercyclical, that is,

$$\frac{\partial \zeta}{\partial \theta} < 0.$$

This implies that equity issuance is procyclical as is illustrated by the following proposition.

Proposition 1 *Suppose that Assumption A holds. Then both debt and equity are procyclical, that is,*

$$\frac{de}{d\theta} > 0 \text{ and } \frac{d(k-n)}{d\theta} > 0 \text{ for } n > 0. \quad (14)$$

The key part of the proof is to show that $\partial\zeta/\partial\theta < 0$, that is, that the shadow price of debt increases during a boom.[21]

The intuition for this result is as follows. The maximization problem indicates that firms face a trade off between higher investment, k, and a lower default probability, $\overline{\omega}$. As θ increases, then the break-even condition of the bank shifts out and it is feasible to have a debt contract with *both* a higher level of k *and* a lower expected default rate. Although feasible, it turns out not to be optimal. The reason is that the break-even condition of the bank not only shifts out but also becomes steeper as θ increases. This means that the relative attractiveness of expansion versus safety, i.e. a higher k versus a lower value for $\overline{\omega}$, shifts towards expansion and an increase in the default rate. It is this increase in $\overline{\omega}$, that is behind the increase in the shadow price of debt when θ increases.[22]

3.4 Pros and cons of the standard debt contract

The standard model of frictions in debt financing implies procyclical equity as well as procyclical debt financing. The discussion above, also brings up the well-known *un*desirable

[21] The proof is given in Appendix D.

[22] This positive relationship between $\overline{\omega}$ and ζ follows almost directly from Equation (8) and Assumption A.

counterfactual feature of the standard debt contract, namely that the default probability, $\overline{\omega}$, is procyclical. In Appendix C, we show that under the standard assumption of diminishing returns, i.e., $\alpha < 1$, that

$$\frac{\partial \overline{\omega}}{\partial x} < 0. \tag{15}$$

That is, an increase in the firm's available level of internal financing decreases the default probability. In a dynamic version of the model, the value of x is procyclical. If the procyclicality of x and the dependence of $\bar{\omega}$ on x are strong enough, then $\bar{\omega}$ would be countercyclical. In this case, equity issuance would be countercyclical. In the previous section, we documented that equity issuance is procyclical for a large set of firms. With acyclical equity issuance costs, the model cannot generate this prediction and at the same time generate a countercyclical default rate. In Section 4, we show that the model can generate both a countercyclical default rate and procyclical equity issuance by relaxing the assumption that equity issuance costs, $\lambda(e)$, are acyclical.

4 Dynamic Model

In this section, we discuss the dynamic version of the model.

4.1 Technology

In addition to making firms forward looking, the dynamic model has some features that are not present in the static model. The first is the specification of the law of motion for productivity. Second, we introduce two minor changes in technology that are helpful in letting the model match some key statistics, such as leverage and the fraction of firms that pay dividends. In particular, we introduce stochastic depreciation and assume that production requires making a periodic fixed production cost.

Productivity. The law of motion for aggregate productivity, θ_t, is given by

$$\ln(\theta_{t+1}) = \ln(\bar{\theta})(1-\rho) + \rho \ln(\theta_t) + \sigma_\varepsilon \varepsilon_{t+1}, \tag{16}$$

where ε_t is an identically, independently distributed (i.i.d.) random variable with a standard normal distribution.

Stochastic depreciation. For constant and low depreciation rates, firms default only for very low realizations of the idiosyncratic shock, because undepreciated capital provides a gigantic safety buffer. A typical reason behind defaults occurring in the real world is that the value of firm assets has deteriorated over time; for example, because the technology has become outdated. To capture this idea, we introduce stochastic depreciation, which makes it possible to generate reasonable default probabilities while keeping the *average* depreciation rate unchanged. In particular, depreciation depends on the same idiosyncratic shock that affects production, and is equal to

$$\delta(\omega_t) = \delta_0 \exp(\delta_1 \omega_t). \tag{17}$$

Fixed production cost. Given the importance of internal funds, it is important to match data on funds being taken out of the firm. For realistic tax rates, profits turn out to be high, which in turn implies that the amount of dividends being paid out is too high. To ensure that the model can match the observed fraction of dividend payers, we introduce a fixed production cost, η. Total output produced is, thus, given by

$$\theta \omega k^{\alpha} - \eta. \tag{18}$$

4.2 Debt and equity contract

At the beginning of the period, aggregate productivity, θ_t, and the amount of cash on hand, x_t, are known. After θ_t is observed, each firm makes the dividend/equity decision and at the same time issues bonds. In the dynamic version, a firm takes into account its continuation value. That is, it maximizes expected end-of-period firm *value*, instead of end-of-period *cash on hand*. Firms default when the amount of funds available at the end of the period is not enough to pay back the debt providers, i.e., when $x_{t+1} < 0$.[23] The

[23] A zero value for cash on hand is the correct default cut-off if firms can default and restart with zero initial funds. We also analyzed the model under the assumption that firms default when $v(x_{t+1}; \theta_{t+1}) < 0$,

debt contract is therefore given by

$$w(n_t;\theta_t) = \max_{k_t,\overline{\omega}_t,r_t^b} \mathrm{E}\Big[\int_{\overline{\omega}_t}^{\infty} v(x_{t+1};\theta_{t+1})d\Phi(\omega) + \int_0^{\overline{\omega}_t} v(0;\theta_{t+1})d\Phi(\omega)|\theta_t\Big] \quad (19a)$$

$$\text{s.t.} \quad x_{t+1} = (1-\tau)[\theta_t\omega_t k_t^{\alpha} - \delta(\omega_t)k_t - \eta - r_t^b(k_t - n_t)] + n_t, \quad (19b)$$

$$0 = (1-\tau)[\theta_t\overline{\omega}_t k_t^{\alpha} - \delta(\overline{\omega}_t)k_t - \eta - r_t^b(k_t - n_t)] + n_t, \quad (19c)$$

$$(1+r)(k_t - n_t) = \int_0^{\overline{\omega}_t}[\theta_t\omega_t k_t^{\alpha} + (1-\delta(\omega_t))k_t - \eta - \mu k_t^{\alpha}]d\Phi(\omega) + (1-\Phi(\overline{\omega}_t))(1+r_t^b)(k_t - n_t). \quad (19d)$$

Note that taxes are a constant fraction of taxable income, which is defined as operating profits net of depreciation and interest expense.

The specification of the equity contract is still given by Equation (10), but $w(\cdot)$ is now given by Equation (19) and equity issuance costs are given by

$$\lambda(e_t;\theta_t) = \lambda_0\theta_t^{-\lambda_1}e_t^2, \quad (20)$$

which allows for the possibility that equity issuance costs are countercyclical ($\lambda_1 > 0$). The idea that equity issuance costs are countercyclical is not new. One reason for positive equity issuance costs is that investors need to be compensated for the firm's incentive to issue equity when its equity is overvalued and Choe, Masulis, and Nanda (1993) argue that this concern is countercyclical. The idea is the following. Firm value is affected by idiosyncratic and aggregate factors. The concern that the firm is exploiting private information is most likely to be related to the idiosyncratic component. Consequently, if aggregate conditions improve, then the idiosyncratic component becomes *relative* to total firm value less important, which in turn reduces the concern of investors to buy overvalued equity.

i.e., when firm value is negative Since $v(0;\theta_{t+1}) > 0$, this means that firms default only when cash on hand is *sufficiently* negative. The model with the alternative specification is more difficult to solve, which makes a careful calibration cumbersome. In those cases where we considered both default specifications, we found the results to be very similar.

Timing. We assume that the period t decisions, including the investment decision, are made when period t productivity is known. This timing is consistent with other papers using this framework to model debt financing.[24] But it deviates from the timing of the standard neoclassical growth model in which the period t capital stock is chosen in period $t-1$, when period t productivity is not yet known.[25] When this timing is used, a positive period-t productivity shock has two effects. First, there is an increase in the expected value of θ_{t+j} for $j \geq 1$. This corresponds to the effect we capture. Second, there is an unexpected windfall in profits. We do not capture this effect. Under this alternative timing, default rates should decrease in the first period, since the amount of debt has been predetermined and revenues are unexpectedly higher. Consequently, by adopting this timing one can make default rates less procyclical without affecting the cyclicality of equity issuance (which is affected by changes in *expected* productivity. Below we will show that we can generate countercyclical default rates, even when this timing is not adopted.

Number of firms. Our model has a fixed number of heterogeneous firms. A firm that defaults on its debt obligations is replaced by a new firm that starts with zero cash on hand.[26]

4.3 Partial equilibrium

Investors who provide funds through debt or equity earn a *constant* expected rate of return equal to r. The rate that firms pay for external finance is time varying and equal to this constant rate plus the endogenous external finance premium, which varies with net worth and aggregate conditions.

A constant expected rate of return would be a general equilibrium outcome if investors are risk neutral. Obviously, this is not a plausible assumption and our model is best interpreted as a partial equilibrium model that abstracts from changes in the required rate of return.

[24] An example would be Carlstrom and Fuerst (1997).

[25] This is also the timing used in Bernanke, Gertler, and Gilchrist (1999).

[26] See Covas (2004) for a model in which the number of firms is determined by a free-entry condition.

Cyclical changes in the required rate of return are likely to amplify the results we emphasize in this paper, for example, because investors require a higher compensation for risk increases during a recession. But a model with endogenous required rates of return would complicate the numerical analysis substantially, because one would have to keep track of the cross-sectional distribution of firms' net worth levels. Adding a time-varying cross-sectional distribution as a state variable to our already complex setting would be quite a challenge. Moreover, to generate realistic time-varying required rates of return would require a lot more than just adding a risk-averse investor to the model.[27]

There is one more reason why our setup is a partial equilibrium analysis. Our model is not appropriate to describe the really large firms, definitely not the top 1% of listed firms. There are several reasons why this is the case. For example, our model does not allow for leveraged buyouts, which are an important aspect behind the financing behavior of the largest firms.[28] As discussed in Section 2, the cyclical behavior of equity issuance of the largest firms is the opposite of that of the other firms. Consequently, there could very well be important general equilibrium effects that we miss with our analysis. For example, if leveraged buyouts by the largest firms reduce equity holdings during a boom, then it may be easier for firms to issue equity so that investors can replenish their equity holdings.

Finally, it must be pointed out that we consider only one type of shock, namely a productivity shock. Although not ideal, this is less problematic here given that we do not have a general equilibrium framework anyway. That is, since we do model neither the consumer side nor the labor market, an increase in productivity is similar to an increase in the price level relative to wages due to, for example, a demand shock.

[27] Boldrin, Christiano, and Fisher (2001) are quite successful in replicating key asset-price properties, but they use preferences that display habit formation, investment that is subject to adjustment costs, multiple sectors, and costs to move resources across sectors.

[28] Another important aspect of firm financing of the largest firms is the occurrance of mergers and the existence of tax incentives to retire equity during mergers.

4.4 Calibration

The parameters fall in one of two groups. In the first group are familiar parameters and its values are set equal to values used in the literature. The parameters in the second group are calibrated to match empirical targets.

Parameter values that are not calibrated. The model period is one year, which is consistent with the empirical analysis. The values of the discount factor, $\beta = (1+r)^{-1}$, the tax rate, τ, the persistence of the aggregate shock, ρ, and the curvature parameter in the production function, α, are set equal to values that are used in related studies. Its values, together with a reference source, are given in the top panel of Table 4.[29]

Calibrated parameter values. The other parameters are chosen to match some key moments. The parameter values and the moments we target are given in the bottom panel of Table 4. Although the parameters determine the values of the moments simultaneously, we indicate in the discussion below which parameter is most influential for a particular moment. In the table, this parameter is listed in the same row as the corresponding moment. The set of targeted moments is as follows:

- The ratio of investment to assets, which is pinned down by the parameter that controls average depreciation, δ_0.
- The fraction of firms that pay dividends, which is pinned down by the production fixed cost, η. Note that the fixed cost affects profitability and, thus, the rate of

[29] The benchmark value of α is equal to 0.70, which exceeds the values typically used in business cycle models with labor. But it is standard to use higher values of α in models without labor. Cooper and Ejarque (2003) use a value equal to 0.7; Hennessy and Whited (2005) estimate α to be equal to 0.551; Hennessy and Whited (2006) estimate α to be equal to 0.693 for small firms and equal to 0.577 for large firms; and Pratap and Rendon (2003) estimate α to be between 0.53 and 0.60. It is easy to show that a problem in which technology is given by $k^{\alpha_k} l^{\alpha_l}$ and the wage is constant is equivalent to a problem in which technology is given by k^{α} with $\alpha = \alpha_k/(1-\alpha_l)$. When the original production function satisfies diminishing returns (for example, because of a fixed factor), then $\alpha < 1$.

return on internal funds. The fixed cost is equal to 17.1 per cent of average aggregate output.

- The default rate, which is pinned down by the bankruptcy cost, μ. Our value of μ is equal to 0.15, which implies that bankruptcy costs are, on average, 2.9 per cent of the value of the defaulting firm, $v(\omega\theta k^{\alpha} + (1 - \delta(\omega))k)$.

- The default premium and leverage, which are pinned down by the volatility of the idiosyncratic shock, σ_{ω}, and the parameter that controls the volatility of depreciation, δ_1. Higher values for σ_{ω} and δ_1 imply less certainty exists about the amount of available funds within the firm, which in turn imply a higher premium on debt finance and lower leverage.

- The volatility of aggregate asset growth, which is pinned down by the standard deviation of the innovation to aggregate productivity, σ_{ε}. We use the volatility of aggregate asset growth instead of aggregate output, because we want to target a measure of real activity for the Compustat universe of firms used to calculate the financing flows. Asset growth is within Compustat the best real activity measure available. We also checked whether the volatility of aggregate output in the model is close to the volatility of output using the deflated series for value added of the non-financial corporate sector, which is published by the BEA. The standard deviation of aggregate output in our model turns out to be equal to 0.0336, which is close to the observed volatility of 0.0313 over the period from 1971 to 2004 using the BEA series.

- An average value and a standard deviation for equity issuance costs, which are pinned down by λ_0, the scaling parameter in the equity issuance cost function, and λ_1, the parameter that controls the variation in the cost of issuing equity. The target for the average is 5.7 per cent and the target for the standard deviation is 1.0 per cent. The motivation for these two targets is given in the remainder of this section.

Equity issuance costs targets. For most of the moments it is straightforward to choose (and find) an appropriate empirical variable and we discuss our choices in Appendix A. It is less clear what information to use to calibrate the parameters of the equity issuance costs function. We would like to use a measure for the average equity issuance cost and a measure for its volatility.

Our targets are based on information on direct costs (underwriting fees) documented in Kim, Palia, and Saunders (2003, 2005). They report an average underwriting spread of 7.6 per cent for initial public equity offerings (IPOs), and 5.1 per cent for seasoned public equity offerings (SEOs). Our target for *average* equity issuance cost is set equal to 5.7 per cent, which is a weighted average of the observed direct costs for IPOs and SEOs using the observed volumes of IPOs and SEOs from Compustat to construct the weights. By basing our calibration only on direct costs we clearly are not overestimating the importance of equity issuance costs.[30]

Kim, Palia, and Saunders (2003) report that several macroeconomic variables are significant in explaining changes in firm level equity issuance costs, but business cycle variables are not among the macro variables considered. A visual inspection of the graph of quarterly means and medians for indirect costs, however, reveals sharp increases in the early eighties, early nineties, and the beginning of the millennium, that is, during economic downturns. Using the time series provided in Kim, Palia, and Saunders (2005) for average direct costs, we calculate the standard deviation of the cross-sectional average of direct costs for IPOs to be equal to 1.23 and for SEOs to be equal to 0.69 per cent. As our target we use a value of 1.0, which is in between these two numbers. The value of λ_1 that generates is equal to 20. Given that the target of 1.0 is only based on the volatility of direct equity issuance costs and ignores indirect costs like underpricing, it may very well

[30] Using the difference between the closing and the offer price to construct an estimate of indirect costs, Kim, Palia, and Saunders (2003) report an average of 31.2 per cent for IPOs and 2.6 per cent for SEOs. They also report a wide range of different values. When the lowest and highest 5 per cent are ignored, then the indirect cost varies from -6 per cent to 156 per cent for IPOs, and from -4.7 per cent to 13.1 per cent for SEOs. Similarly, Loughran and Ritter (2002) report that $9.1 million "is left on the table" for the average IPO, which corresponds to three years of operating profits.

be too low. Therefore, we also consider $\lambda_1 = 40$ and $\lambda_1 = 60$.

5 Cyclicality of debt and equity in the model

In this section, we discuss the cyclical patterns of debt and equity issuance in the dynamic model. In Section 3, we showed analytically that equity issuance is procyclical if debt financing is modeled using the standard framework with bankruptcy costs and idiosyncratic productivity shocks. There are several questions that remain to be answered. First, is it possible to have procyclical equity financing and a countercyclical default probability? We will see that this is possible if the friction of issuing equity is sufficiently countercyclical. Second, does the cyclical behavior of equity issuance depend on firm size as it does in the data? The answer is yes. Third, what are the quantitative predictions of the model in terms of the cyclical responses of debt and equity issuance and in terms of magnification? As discussed in more detail below, the quantitative results are somewhat disappointing.

We start with a discussion of the responses of debt, equity, dividends, and net worth following an aggregate productivity shock. Next, we discuss key summary statistics such as correlation coefficients and variances. To correctly interpret our results it is important to realize that equity issuance and dividends are by definition non-negative. That is, we do not refer to dividends as negative equity issuance.

5.1 Cyclicality of debt and equity issuance: IRFs

Figure 1 plots the Impulse Response Functions (IRFs) of debt, equity, net worth, and dividends for a positive one-standard-deviation aggregate productivity shock. As discussed above, we are mainly interested in the results for the bottom 90%. To streamline the discussion we plot the IRFs for only two firm groups, namely the firm group with the smallest and the firm group with the largest firms in the bottom 90%.[31]

[31] We exclude the largest firms, since our model is not appropriate to explain the largest firms. In the model the firms in the top 10% behave very similarly as those in the [75%, 90%] firm group, in contrast to the data.

Equity. The results for equity are straightforward. Small firms respond to the productivity shock by sharply raising additional equity. The higher the value of λ_1, i.e., the more countercyclical the friction of issuing equity, the stronger the response. Firms in the $[75\%, 90\%]$ firm group do not raise funds through equity contracts, not during normal times and not during expansions. Consequently, their IRF of equity is, thus, equal to zero. Below, we will discuss in more detail the size dependence of equity issuance and document that firms in the $[50\%, 75\%]$ firm group do issue more equity in response to a productivity shock.

Dividends. Small firms do not issue dividends; consequently their IRF of dividends is equal to zero. Large firms do issue dividends. Consider the IRF for dividends issued by the $[75\%, 90\%]$ firm group when λ_1 is equal to 20. Following a productivity shock, dividends fall sharply in the first period. In the second period, the response reverses and dividends take on values that are substantially higher than their pre-shock levels. For the two higher values of λ_1 considered, namely 40 and 60, the drop in the first-period is muted, but the positive response of dividends in the subsequent periods is still present. Thus, our model has the remarkable prediction that both aggregate equity and aggregate dividends increase following a positive productivity shock (except for a change in dividends in the opposite direction in the first period). A model with a representative firm could never generate such a result. Our model with heterogeneous firms can. We will now shed some light on the factors driving this result.

The initial decrease in dividends is consistent with Proposition 3, according to which $de/d\theta > 0$. That is, the increase in the shadow price of debt induces some firms to stop issuing dividends and raise equity instead. In subsequent periods, dividends increase because net worth increases following the productivity shock. That is, increased profitability raises net worth, which means that more firms get into the range where it becomes attractive to pay dividends. But there is another factor that puts upward pressure on dividends and this channel also explains why the initial decrease in dividends is smaller for higher values of λ_1, i.e., when the cost of issuing equity is more countercyclical. Firms that issue dividends are not directly affected by the decrease in equity issuance costs when productivity increases. Nevertheless, the generated first-period drop in equity strongly depends on the value of λ_1. The reason is that these firms may want to (or need to) issue equity in the future. Firms maintain their current net worth as a buffer against this possibility, since it is costly to issue equity. The need for such a buffer is less when equity issuance costs are expected to be less in the future.

Net worth. The net worth IRF for small firms is hump shaped, which is consistent with the textbook shape according to the net worth channel. That is, an increase in net worth leads to a reduction in the bankruptcy friction, which in turn leads to an increase in external funds and a further increase in net worth even though productivity has already started to decline along its mean reverting path. Not surprisingly, we find that the increase in net worth for small firms is larger when the decrease in the cost of issuing equity is stronger, that is, when the value of λ_1 is higher.

The IRFs for the firm group with the largest firms is not hump shaped. Propagation through a net worth channel is apparently not important for these firms. The reason is that financial frictions are not that important for large firms. Interestingly, for large firms the increase in net worth is *smaller* for higher values of λ_1. The role of net worth as a buffer against future negative shocks is important to understand this result. Following a persistent productivity shock, firms expect a period in which equity can be raised cheaply, which reduces the need to maintain a high net worth level for insurance purposes. This effect is more important for higher values of λ_1.

Debt. All firms expand in response to a productivity shock and all firms finance this at least in part by taking on additional debt. The debt responses for small firms are smaller than the debt responses for large firms. The opposite is true for equity suggesting that debt and equity are substitutes. To some extent this is, of course, the case. But the two types of financing can also reinforce each other. To see why this is the case, consider the effect of an increase in λ_1 on the debt and equity responses for small firms. When λ_1 increases, then the equity responses increase. This is not surprising, since the higher the value for λ_1 the larger the reduction in equity issuance costs when productivity increases.

But this higher increase in equity does not lead to a smaller increase in debt. Instead, with the additional amount of funds raised through equity finance firms also raise more funds through debt financing.

5.2 Cyclicality of debt and equity issuance: correlation coefficients

Table 2 reports the correlation coefficients of HP-filtered output and the two external financing sources. Debt issuance basically follows the productivity shock. Consequently, the correlation coefficients for debt and output in our one-shock model are all close to 1.

For equity, the correlation coefficients are zero for those firm groups that never issue equity and close to zero for those firm groups in which few firms issue equity. But for firms in the three firm groups in the bottom 75% equity issuance is clearly procyclical as it is in the data. As discussed above and documented in Figure 1, the response of equity following a productivity shock is positive in every single period. The reason the correlation coefficients are not equal to one is because the dynamics of equity issuance do not exactly follow those of the productivity shock. In particular, the IRF of equity issuance returns to zero quicker than the IRF of productivity, especially when the value of λ_1 is low. The reason is that aggregate productivity also increases internal finance and leads to more rapid firm growth. Since larger firms issue less equity, there also is a downward effect on equity issuance. This never leads to negative values of the IRF, but does lead to smaller responses *relative* to those of the productivity shock at least when λ_1 is small. This explains why the correlation coefficient for small firms is only equal to 0.17 for firms in the bottom quartile when $\lambda_1 = 0$. Note that this correlation coefficient shoots up to 0.43 when $\lambda_1 = 20$, which is not much less than the value of 0.53 that is observed in the data.

5.3 Cyclicality of debt and equity issuance: Volatilities

Correlation coefficients do not provide information with how much debt and equity issuance change over the business cycle. To shed light on this we plot in Figure 2 the values of $\frac{\Delta D}{A_{-1}}$ and $\frac{\Delta E}{A_{-1}}$ in the first period of the shock together with the corresponding panel regression estimates from Covas and Den Haan (2010a). The conclusion is that the model does a good job in replicating the qualitative aspects of the observed cyclicality of debt and equity issuance, but fails to match the quantitative aspects in some important dimensions.

First, consider the results for debt displayed in the bottom panel. The model correctly predicts that debt is procyclical and that there is limited size dependence. The model overpredicts the magnitude of the debt fluctuations over the cycle and this is true for all values of λ_1 considered.

Next, consider the results for equity displayed in the top panel. The model correctly predicts that equity is procyclical and that the magnitude of the cyclical changes are larger for small firms. But the size dependence predicted by the model is too strong and the predicted magnitude of the cyclical changes is too small as well. Only for the highest value of λ_1 considered, i.e., $\lambda_1 = 60$, is the model capable of matching the observed response of equity issuance for the bottom 25%. For the other firm groups, the model underpredicts the magnitude of the cyclical fluctuations in equity issuance, even for the highest value of λ_1. Our calibration procedure was based on changes in the direct cost of equity issuance and resulted in a value of λ_1 equal to 20. The results here indicate that this is a very conservative approach and that cyclical changes in the cost of issuing equity may be much larger.

6 Magnification and propagation in the model

Cochrane (1994) points out that it is difficult to think of external shocks to the economic system that are large enough to generate fluctuations with amplitudes like those observed during regular business cycles let alone during periods of crisis. The challenge is, therefore, to develop models that amplify and propagate shocks. The standard debt contract is in that sense a step backward. As shown in Appendix C.2, the standard debt contract has the unfortunate property that it *dampens* shocks.[32] The reason is that following a decrease in

[32] It is possible to obtain magnification with only debt financing. For example, Bernanke, Gertler, and Gilchrist (1999) introduce capital adjustment cost, which makes the price of capital procyclical. The market

productivity the firm's initial net worth (which is fixed) actually increases *relative* to the optimal size of the firm (which decreases). But this means that the bankruptcy friction actually becomes less important as productivity declines, which in turn means that the effects of the shock are dampened. A related property is that defaults are procyclical in the standard debt problem. A productivity increase leads to an *in*crease in default rates, because of the strong desire of firms to expand when productivity increases. The subsequent increase in net worth levels has no effect at all on the default rates in the standard framework because it assumes that the production function is linear.

In this section, we show that these undesirable properties can be overturned if one allows firms to also attract equity financing. We will first show that the model with equity finance can generate a countercyclical default rate. Next, we will show that there is no dampening in the model. In fact, there is magnification, although the magnification is small except for the small firms and then only when the value of λ_1 is high.

6.1 Generating a countercyclical default rate

In this section, we focus again on the responses to a positive one-standard-deviation productivity shock. Figure 3 plots the responses of the average default rate for firms in the bottom quartile and the average default rate for all firms.[33] The figures show the results for values of λ_1 equal to 0, 20, 40, and 60. The shape of the IRF for small firms is similar to the IRF for the default rate averaged across all firms, except that the increase in the default rate is much higher for small firms.

First consider the case when $\lambda_1 = 0$, that is, when the friction to issue equity is itself acyclical. When $\lambda_1 = 0$, then the default rate is procyclical although not as procyclical

value of the firm's net worth then increases during a boom which relaxes the firm's financing constraints. In our model, the relative price of capital is fixed. The value of firms' net worth still increases because additional equity is being issued and (as in Bernanke, Gertler, and Gilchrist (1999)) through increased profits.

[33] To be consistent, we exclude again the largest firms. Given that default is not an issue for the largest firms in our model, it does not matter for the default rate whether one focuses on the bottom 90%, the bottom 99%, or all firms.

as in the model without equity issuance. Our model dampens the procyclicality of the standard debt contract even when $\lambda_1 = 0$, because with a *nonlinear* production function an increase in net worth lowers bankruptcy. Net worth levels increase because profits increase and because equity issuance increases. When $\lambda_1 = 0$, equity issuance increases because the shadow price of debt increases, which is directly related to the increase in default rates. Thus, the increase in the default rate increases the shadow price of debt, which in turn increases equity issuance. The latter dampens the increase in the default rate, but can never overturn it because if the default rate were to decrease equity issuance should decrease as well. Thus, this direct increase in net worth through extra equity financing triggered by the increase in productivity cannot lead to a reduction in default rates. But in our model with a standard nonlinear production function, the *subsequent* increase in net worth due to higher profit levels could in principle accomplish this. However, for our calibrated parameter values this effect is not strong enough to get default rates to decrease.

In contrast, for all three positive values of λ_1 considered, the model predicts a decrease in the default rate following a positive productivity shock. When $\lambda_1 = 20$ this decrease is only 5 basis points for all firms and only 20 basis points for firms in the bottom 25%, but when $\lambda_1 = 60$, these two numbers are equal to 27 and almost 120 basis points. It is difficult to determine the correct empirical analogue given that the cyclicality of default rates seems to have increased over time even though cyclical changes in output became smaller.[34] Relative to the increases in the default rates observed during the recession of the early nineties and the recession at the beginning of the Millennium, which are increases of several percentage points, the fluctuations generated by the model seem moderate, even when λ_1 is equal to 60.[35]

6.2 Magnification through equity issuance

Figure 4 plots the IRF of output in (i) the model in which firms do not face any restrictions in obtaining firm finance and (ii) the model in which firms only have access to debt finance

[34] The small number of rated firms by Moody's in the 1980s and the emergence of junk bonds in the early eighties are likely to have been important for this change.

[35] See Appendix B for further details on the cyclical behavior of the default rate on corporate debt.

and debt finance is subject to the bankruptcy friction. The standard nonlinear production function is used. Thus, firm size matters. In particular, firms with lower net worth levels have higher average default rates. The top panel plots the results for the bottom quartile and the bottom panel plots the results for the $[75\%, 90\%]$ firm group. The bottom panel shows that the IRFs of the two models are indistinguishable for large firms. This makes sense. If the net worth level of a firm is high enough, then it is no longer affected by the bankruptcy friction and it responds to shocks in the same way as a firm in the frictionless model. In contrast, the IRFs of the two models do differ for small firms. Consistent with the reasoning given at the beginning of this section on page 25, the response of small firms is actually *smaller* in the model in which firms rely on debt financing and the friction is the standard bankruptcy friction. That is, frictions dampen the consequences of the shocks. Although introducing the financial friction into the model worsens the ability of the model to magnify shocks, the dampening introduced is fairly small.

Figure 5 plots the corresponding IRFs for our model with equity issuance for different values of λ_1 and it also includes the responses for the frictionless case. For large firms, output responses are again not very different from those of the frictionless model. For small firms, however, we find that the output responses are quite different from the responses in the frictionless model. For all three non-zero values of λ_1 considered, we find that the output responses are stronger than the responses of the frictionless case. When $\lambda_1 = 60$, then we find that the first-period response in the model with equity issuance is 1.26 percentage points above the response of the frictionless model and 1.46 percentage points above the response of the model in which firms can only issue debt and face the standard bankruptcy friction. It must be noted that there is substantially less magnification for smaller values of λ_1. That is, magnification is possible but it does require a substantial amount of time variation in the frictions firms face in raising equity.

7 Conclusions

Most models used to study the role of frictions in obtaining firm finance for business cycles assume that firms can obtain external financing *only* through a one-period debt contract. But firms use other forms of financing and, in particular, they rely on equity. A proper study of the role of financial frictions should take this into account and it is therefore important that theoretical challenges to study this more complex environment are overcome.

In this paper, we start with a commonly used framework in which firms can use only debt finance and the friction is an unavoidable distress state with deadweight losses if the firm's resources are not enough to pay back the debt. Next, we introduce the ability to raise equity and assume that the friction to raise equity is itself acyclical.

This simple setup allows us to bring to the surface some of the forces affecting the cyclical behavior of firm finance. In particular, equity issuance is procyclical—even if equity issuance costs are acyclical—*if* the friction to obtain debt finance is procyclical. This is indeed the case in this framework, because the strong desire to expand to take advantage of the high productivity leads to an *increase* in the default rate on debt. It is possible for the model to generate both procyclical equity finance and a countercyclical default rate, but only if the friction in obtaining equity finance is sufficiently countercyclical. Another insight that the model provides is that the introduction of equity issuance can overcome the undesirable feature of the debt-only model that shocks are being dampened. Quantitatively the effects are only substantial for small firms and only if the friction in obtaining equity finance changes sharply over the cycle.

Possibly the most important contribution of this paper is that it makes clear that still a lot of work remains to be done. Based on our work in this area, we think that work in the following three directions would be very useful.

The disadvantage of the way we model the countercyclicality of equity issuance costs is that it is ad hoc. The advantage is that the analysis remains transparent. We think that the advantage outweighs the disadvantage, especially since so little has been done in terms of modeling equity finance. Explicit modelling of the friction firms face in obtaining firm finance would be an important step forward. Obviously, this would be a non-trivial exercise.

The second direction in which the theory needs to be extended is to include the largest firms into the analysis. The empirical work has made clear that the cyclical behavior of equity issuance is very different for the largest firms. Moreover, in reality the largest firms are so large and the cyclical behavior of equity issuance so different, that the results for observed aggregate equity issuance can switch from countercyclical to procyclical if these largest firms are excluded. Thus, one needs a model that can generate a right tail with very large firms that behave very differently. A key element in the observed financing behavior of the largest firms is that they are typically highly rated and have a direct access to market debt which may be used to finance share repurchases in good times. In addition, mergers, acquisitions and leveraged buyouts are highly procyclical and often lead to net equity retirements. Thus, it is not too surprising that net equity issuance by the largest firms is countercyclical. A model that allows for these feautures may be able to generate both the observed skewed distribution for firm size and the observed different cyclical behavior for equity issuance by the largest firms.

Finally, it would also be important to extend the model to allow for cyclical changes in the required expected rate of return by investors. Covas and Den Haan (2006) document that the model can generate more sizable and realistic fluctuations in equity issuance if the rate of return that investors require to invest in equity is countercyclical. Moreover, with this feature the model also does a better job in magnifying shocks. The disadvantage of the approach in Covas and Den Haan (2006) is that it is simply assumes that the required rate of return is cyclical. What is needed is an equilibrium model in which such cyclical changes are endogenous. It would be very interesting to use such a model to analyze the feedback effects between cyclical changes in required rates of return and the consequences of these for the business cycle through its effects on the ability of firms to raise external finance.

A Data Sources

Compustat. Our data are taken from Compustat and consists of annual data from 1980 to 2006. In the appendix of Covas and Den Haan (2010a) it is shown that the results are robust to including the earlier part of the data set. Compustat includes firms listed on the three U.S. exchanges, NYSE, AMEX, and Nasdaq, with a non-foreign incorporation code. We exclude financial firms (SIC codes 6000-6999), utilities (SIC codes 4900-4949), and firms involved in major mergers (Compustat footnote code AB) from the whole sample.[36] We also exclude firms with a missing value for the book value of assets, and firm-years that violate the accounting identity by more than 10 per cent of the book value of assets. Finally, we eliminate the firms most affected by the accounting change in 1988, namely GM, GE, Ford, and Chrysler.[37]

The firms included form an important part of the U.S. economy, not only in firm assets, but also in terms of employment. We have employment numbers for 94 per cent of our firms; total employment for these firms is equal to 35 million, which is roughly one quarter of total U.S. employment.

For equity issuance we use the net change in the book value of equity, i.e., stockholders' equity (item #216). This series excludes accumulated retained earnings. For debt issuance we use the net change in the book value of total liabilities (item #181). See Covas and Den Haan (2010a) for further details.

Output and deflator. The financing series are deflated with the producer price index for industrial commodities from the Bureau of Labor Statistics. Real GDP is real gross domestic product of the corporate sector, chained 2000 billions of dollars, from the Bureau of Economic Analysis.

Default rate and premium. The annual default rate is from Moody's (mnemonic USMDDAIW in Datastream), and it is for all corporate bonds in the United States. The default premium is the estimated default spread on corporate bonds taken from Longstaff, Mithal, and Neis (2005).

[36] Compustat assigns a footnote AB to total sales if sales increase by more than 50 percent in response to a merger or an asset acquisition. If the firm has been involved in a merger or acquired assets, but total sales did not increase by more than 50 percent, than this firm is still included in our sample.

[37] See Bernanke, Campbell, and Whited (1990) for details.

B Default rate

Figure 6 plots the default rate on corporate bonds. The figure documents that the default rate is clearly countercyclical. It also shows that the cyclical fluctuations have become a lot stronger starting in the early eighties. The emergence of junk bonds is likely to be important for this change.

C Properties of the standard debt contract

C.1 Response of the default rate when $\alpha = 1$ and when $\alpha < 1$

In this section, we show how the default rate, $\overline{\omega}$, changes when net worth and productivity increases. The analysis brings to the surface the restrictive nature of the commonly made assumption that the production function is linear, i.e., the case with $\alpha = 1$. In particular, whereas the default rate does not depend on net worth when α is equal to 1, the default rate decreases with net worth for the more regular case with $\alpha < 1$. The following proposition summarizes the key properties of the default rate.

Proposition 2 *Suppose that Assumption A holds. Then,*

$$\begin{aligned} \frac{d\overline{\omega}}{dn} &= 0 \text{ when } \alpha = 1, \\ \frac{d\overline{\omega}}{dn} &< 0 \text{ when } 0 < \alpha < 1, \text{ and} \\ \frac{d\overline{\omega}}{d\theta} &> 0 \text{ when } n > 0 \text{ and } 0 < \alpha \leq 1 \\ \frac{d\overline{\omega}}{d\theta} &= 0 \text{ when } n = 0, \text{ and } 0 < \alpha < 1. \end{aligned}$$

The proofs of the proposition are given in Appendix C.3. The first two parts of the proposition say that an increase in the firm's net worth has no effect on the default rate when technology is linear (i.e., $\alpha = 1$), but reduces the default rate when technology exhibits diminishing returns (i.e., $\alpha < 1$). This is an interesting result, since it makes clear that when $\alpha = 1$, i.e., the case considered in the literature, an increase in net worth, which is the key variable of the net-worth channel, does *not* lead to a reduction in the default rate. In particular, Levin, Natalucci, and Zakrajsek (2004) analyze the case with $\alpha = 1$ and document—using an estimated version of the model—that observed changes in idiosyncratic volatility and observed changes in leverage caused by changes in the value of net worth cannot generate substantial changes in the external finance premium. Thus, if changes in net worth are to have a substantial effect on the default probability and the finance premium, it is essential to adopt a value of α that is less than 1.

The last two parts of the proposition say that an increase in aggregate productivity increases the default rate, except when $n = 0$.[38] That is, an increase in θ changes the firm's trade-off between expansion (higher k) and less defaults (lower $\overline{\omega}$) in favor of expansion. With $\alpha = 1$, an increase in θ therefore leads to an increase in the default rate and any subsequent increase in net worth would not affect it. With $\alpha = 1$ and without further modifications, dynamic models with the standard debt contract, thus, generate a procyclical default rate, which is counterfactual.[39] With $\alpha < 1$, the increase in n that follows an increase in θ does have a considerable downward effect on the default rate, but we never find this effect to be large enough to overturn the effect of the increase in θ.

C.2 Dampening instead of propagation with the standard debt contract

Cochrane (1994) argues that there are few external sources of randomness that are very volatile. The challenge for the literature is therefore to build models in which small shocks can lead to substantial fluctuations. The debt contract has the unfortunate property that it dampens shocks. That is, the responses of real activity and capital in the model with the debt contract are actually less than the responses when there are no frictions in obtaining external finance. This is summarized in the following proposition. Let y be aggregate output and let y^{net} be aggregate output net of bankruptcy costs. Also, let $\widetilde{k}$ and $\widetilde{y}$ be the

[38] The last part of the proposition imposes that $\alpha < 1$, because when $\alpha = 1$ the problem is not well defined for $n = 0$.

[39] To alleviate this problem, Bernanke, Gertler, and Gilchrist (1999) assume that aggregate productivity is not known when the contract is written. Dorofeenko, Lee, and Salyer (2006) generate a countercyclical default rate by letting idiosyncratic risk decrease with aggregate productivity.

solution to capital and aggregate output in the model without frictions, respectively.

Proposition 3 *Suppose that $n > 0$ and Assumption A holds. Then,*

$$\frac{d\ln k}{d\ln\theta} < \frac{d\ln\widetilde{k}}{d\ln\theta} = \frac{1}{1-\alpha}, \text{ and} \tag{21}$$

$$\frac{d\ln y^{net}}{d\ln\theta} < \frac{d\ln y}{d\ln\theta} < \frac{d\ln\widetilde{y}}{d\ln\theta} = \frac{\alpha}{1-\alpha}. \tag{22}$$

To understand this proposition, it is important to understand that beginning-of-period net worth, n, is fixed. For example, consider an enormous drop in θ. Suddenly, n becomes very large relative to θ, but this means that frictions are much less important. The reduction of the agency problem implies that the effect of the drop in θ is reduced. Essential for generating an increase in n relative to θ is, of course, that $n > 0$. The proof in Appendix C.3 makes it clear that if $n = 0$, the percentage changes in capital and output are equal to those of the frictionless model.

C.3 Proofs of propositions 2 and 3

Preliminaries. Before we give the proofs of the propositions, we give the formulas for the derivatives and present a lemma.

The first and second derivatives of $F(\overline{\omega})$ are given by

$$\begin{aligned} F'(\overline{\omega}) &= -(1-\Phi(\overline{\omega})) \leq 0 \text{ and} \\ F''(\overline{\omega}) &= \Phi'(\overline{\omega}) \geq 0. \end{aligned}$$

The first and second derivatives of $G(\overline{\omega})$ are given by

$$\begin{aligned} G'(\overline{\omega}) &= -F'(\overline{\omega}) - \mu\Phi'(\overline{\omega}) \text{ and} \\ G''(\overline{\omega}) &= -F''(\overline{\omega}) - \mu\Phi''(\overline{\omega}). \end{aligned}$$

The signs of the two derivatives of $G(\overline{\omega})$ are not pinned down. For example, there are two opposing effects of an increase of $\overline{\omega}$ on $G(\overline{\omega})$. First, an increase in $\overline{\omega}$ reduces $F(\overline{\omega})$, i.e., the share that goes to the borrower. This corresponds to an increase in lending rates and, thus, an increase in revenues from firms that do not default. Second, an increase in

$\overline{\omega}$ implies an increase in bankruptcy costs. For internal optimal values for $\overline{\omega}$, however, we know that $G'(\overline{\omega}) \geq 0$. If not, then the bank could increase its own and firm profits by reducing $\overline{\omega}$. We summarize this result in the following lemma.

Lemma 1 *For internal optimal values of $\overline{\omega}$, $G'(\overline{\omega}) \geq 0$.*

Lemma 2 *Under Assumption A,*

$$\frac{\partial \left(-\frac{F'(\overline{\omega})}{G'(\overline{\omega})}\right)}{\partial \overline{\omega}} > 0.$$

This is a straightforward implication of Assumption A. To make the algebra less tedious, we set without loss of generality $\delta = 1$ and $r = 0$ in the remainder of this section.

Intuition for proposition 2. Both an increase in k and a reduction in $\overline{\omega}$ lead to an increase in firm profits, and both lead to a reduction in bank profits, at least in the neighborhood of the optimal values for k and $\overline{\omega}$.[40] To satisfy the bank's break-even condition, the firm, thus, faces a trade-off between a higher capital stock and a lower default rate.

If $\alpha = 1$, then the problem is linear and an increase in n simply means that the scale of the problem increases. Consequently, an increase in n does not affect the default rate, but simply leads to a proportional increase in k. When $\alpha < 1$, decreasing returns imply that an increase in k is not as attractive anymore, and the firm will substitute part of the increase in k for a reduction in $\overline{\omega}$ when n increases.

Next, consider what happens if aggregate productivity increases. For the firm, the relative benefit of a higher capital stock versus a lower default rate does not change.[41] An increase in θ means, however, that the break-even condition for the bank becomes steeper; that is, because the bank's revenues in case of default increase, capital becomes cheaper

[40] At very low levels of k, the marginal product of capital is very high and bank profits may be increasing in k. Such low levels of k are clearly not optimal since an increase in k would then improve both firm and bank profits.

[41] That is, the iso-profit curve does not depend on aggregate productivity.

relative to $\overline{\omega}$. In other words, when aggregate productivity is high, then this is a good time for the firm to expand, even when it goes together with a higher default rate.[42]

Proof of proposition 2. The result that $d\overline{\omega}/dn = 0$ when $\alpha = 1$ follows directly from the first-order condition (7). Next, consider the case when $\alpha < 1$. Rewriting the first-order condition gives

$$\frac{1}{\alpha\theta k^{\alpha-1}} = -\frac{G'(\overline{\omega})}{F'(\overline{\omega})}F(\overline{\omega}) + G(\overline{\omega}) \tag{23}$$

$$= \left(1 - \frac{\mu\Phi'(\overline{\omega})}{(1-\Phi(\overline{\omega}))}\right)F(\overline{\omega}) + G(\overline{\omega}). \tag{24}$$

Assumption A, together with Lemma 1, implies that the right-hand side decreases with $\overline{\omega}$. Suppose, to the contrary, that $d\overline{\omega}/dn > 0$. Then, equation (24) implies that an increase in net worth must lead to a decrease in capital. But an increase in $\overline{\omega}$ and a decrease in k reduces expected firm profits, and this can never be optimal, because the old combination of $\overline{\omega}$ and k is still feasible when n increases. Similarly, $d\overline{\omega}/dn = 0$ is not optimal; according to equation (24), it implies that $dk/dn = 0$, but the zero-profit condition of the bank makes an increase in k feasible. Consequently, $d\overline{\omega}/dn < 0$.

We next show that $d\overline{\omega}/d\theta > 0$. By combining equations (6) and (7), we obtain the following expression that does not depend on θ:

$$-\frac{G'(\overline{\omega})}{F'(\overline{\omega})}\frac{F(\overline{\omega})}{G(\overline{\omega})} = \left(\frac{1}{\alpha(1-\frac{n}{k})} - 1\right). \tag{25}$$

This equation immediately proves the last part of the proposition that $d\overline{\omega}/d\theta = 0$, when $n = 0$. Using Lemmas 1 and 2 together, with the result that $F'(\overline{\omega}) \leq 0$, implies that the

[42] In itself this may not be an implausible or undesirable outcome, but it would be if it leads to procyclical default rates, which is counterfactual. With $\alpha = 1$ that would indeed happen. With $\alpha < 1$ an increase in net worth reduces the default rate. Consequently, it is possible that subsequent increases in net worth through retained earnings (that would occur in the dynamic version of the model) would compensate for the upward pressure on the default rate caused by the increase in aggregate productivity. In our numerical experiments, however, we find that the direct effect of the increase in aggregate productivity is substantially stronger.

left-hand side is decreasing in $\overline{\omega}$. The right-hand side is decreasing in k. Thus, k has to move in the same direction as $\overline{\omega}$. A decrease in $\overline{\omega}$ and k, however, is not consistent with (24).[43] ■

Proof of proposition 3. Let $\widetilde{k}$ be the solution of capital when there are no frictions. This capital stock is given by

$$\widetilde{k} = \left(\frac{1}{\alpha\theta}\right)^{1/(\alpha-1)}, \tag{26}$$

which gives

$$\frac{d\widetilde{k}}{\widetilde{k}} = \frac{1}{1-\alpha}\frac{d\theta}{\theta}.$$

From the break-even condition of the bank we get

$$k^{\alpha}G\left(\overline{\omega}\right)d\theta + \theta\alpha k^{\alpha-1}G\left(\overline{\omega}\right)dk + \theta k^{\alpha}G'\left(\overline{\omega}\right)d\overline{\omega} = dk. \tag{27}$$

Using the break-even condition, this can be written as

$$\frac{k-n}{\theta}d\theta + \alpha\frac{k-n}{k}dk + \frac{k-n}{G\left(\overline{\omega}\right)}G'\left(\overline{\omega}\right)d\overline{\omega} = dk, \quad \text{or} \tag{28}$$

$$\frac{d\theta}{\theta} + \alpha\frac{dk}{k} + \frac{G'\left(\overline{\omega}\right)}{G\left(\overline{\omega}\right)}d\overline{\omega} = \frac{k}{k-n}\frac{dk}{k}, \quad \text{or} \tag{29}$$

$$\frac{dk}{k} = \frac{\frac{d\theta}{\theta} + \frac{G'(\overline{\omega})}{G(\overline{\omega})}d\overline{\omega}}{\frac{k}{k-n} - \alpha}. \tag{30}$$

First, suppose that $n = 0$. The denominator is then equal to the denominator in the expression for the case without frictions. From proposition 1, we know that $d\overline{\omega}/d\theta = 0$ if $n = 0$. Consequently, the percentage change in capital in the model with frictions is equal to the percentage change in the model without frictions. When $n > 0$, there are two factors that push in opposite directions. The denominator is now larger than $1-\alpha$, which dampens the increase in capital relative to the increase in the frictionless model. The increase in $\overline{\omega}$, however, implies an increase in $G(\overline{\omega})$, which makes capital more responsive

[43] An increase in θ and a reduction in k lead to a decrease in the left-hand side, while a reduction in $\overline{\omega}$ leads to an increase in the right-hand side.

relative to the increase in the frictionless model. We will next show that the first effect dominates. The first-order conditions are given by

$$\zeta(\overline{\omega}) = \frac{\alpha\theta k^{\alpha-1}F(\overline{\omega})}{1-\alpha\theta k^{\alpha-1}G(\overline{\omega})}, \tag{31}$$

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1-\mu\Phi'(\overline{\omega})/(1-\Phi(\overline{\omega}))}. \tag{32}$$

Let

$$X(\theta,k) = \alpha\theta k^{\alpha-1}. \tag{33}$$

From (31) we get

$$\begin{aligned} FdX + XF'd\overline{\omega} &= \zeta' d\overline{\omega} - X\zeta G' d\overline{\omega} - XG\zeta' d\overline{\omega} - \zeta G dX, \\ (F+\zeta G)dX &= (1-XG)\zeta' d\overline{\omega} + X(1-\Phi-\zeta(1-\Phi-\mu\Phi'))d\overline{\omega}, \end{aligned}$$

$$(F+\zeta G)dX = (1-XG)\zeta' d\overline{\omega} + 0. \tag{34}$$

Lemma 2 implies that $\zeta' > 0$. From (31) we know that $(1-XG) > 0$. Equation (34) then implies that dX and $d\overline{\omega}$ must have the same sign. From proposition 1, we know that $d\overline{\omega}/d\theta > 0$. Thus, according to equation (34), $dX/d\theta > 0$. In the model without frictions, $dX/d\theta = 0$, since without frictions $X = \alpha\theta k^{\alpha-1}$ is constant. But $dX > 0$ implies that $dk/d\theta < d\widetilde{k}/d\theta$.■

D Proof of proposition 1; the procyclicality of debt and equity issuance

Given the preliminary work in Section C, it is now relatively straightforward to prove that in an environment in which firms face the standard debt contract, with the friction of an unavoidable bankruptcy cost in case of default, and an acyclical quadratic cost of issuing equity that both debt and equity respond positively to an increase in productivity.

Key in proving this proposition is the first-order condition of the equity-issuance problem, equation (12). Since equity issuance costs do not depend on aggregate productivity, equity issuance decreases (increases) in response to an increase in aggregate productivity, θ, when $\partial w/\partial e$ decreases (increases) with θ. The marginal value of an extra unit of equity in the firm, $\partial w/\partial e$, is equal to $\zeta(\overline{\omega})(1+r)$. From equation (8) we know that the Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone. Moreover, the regularity condition in Assumption A guarantees that $\zeta(\overline{\omega})$ is increasing in $\overline{\omega}$, which means that the marginal value of an extra unit of equity, $\partial w/\partial e$, is increasing in $\overline{\omega}$. Since $\overline{\omega}$ is increasing with aggregate productivity, $\partial w/\partial e$ is increasing with aggregate productivity, which means that equity issuance is increasing. Thus, an increase in θ increases the default rate, which increases the value of an extra unit of net worth in the firm, $\partial w/\partial e$, which, in turn, increases equity issuance.

It remains to be shown that

$$\frac{d(k-n)}{d\theta} > 0,$$

that is, the increase in net worth induced by the increase in equity finance does not lead to reduction in debt financing. From the lender's break-even condition, we get

$$k - n = \frac{\theta k^{\alpha} G(\overline{\omega})}{\delta + r} + \frac{1-\delta}{\delta + r} n. \tag{35}$$

The last term on the right-hand side captures the leverage affect, that is, firms use the extra net worth to increase their debt levels. Moreover, the increase in $\overline{\omega}$ leads to an increase in $G(\overline{\omega})$.[44] Since θ and k also increase, the right-hand side increases.■

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[44] In principle it is possible that $\partial G(\overline{\omega})/\partial \overline{\omega} < 0$, but it would never be optimal to choose a level of $\overline{\omega}$ at which a decrease could increase the revenues for both the firm and the bank.

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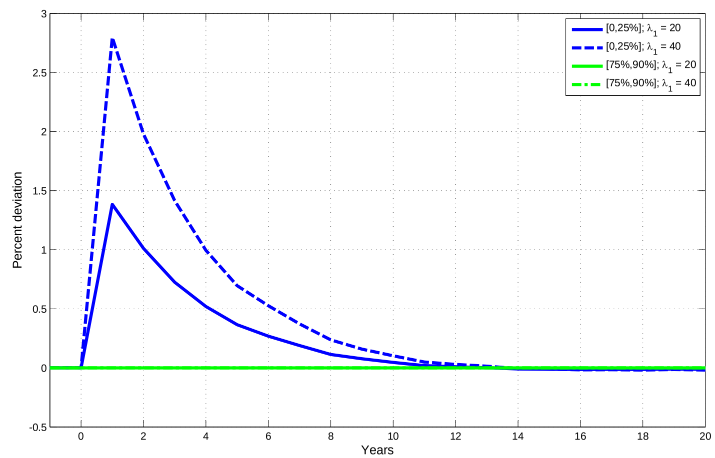
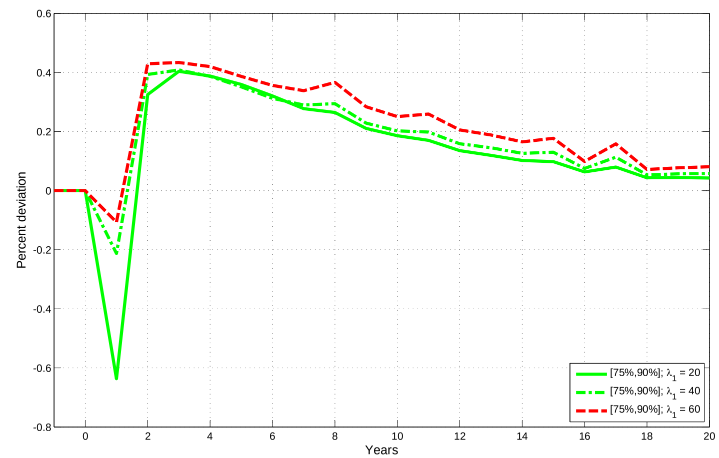
Figure 1: Firm finance responses to a positive productivity shock

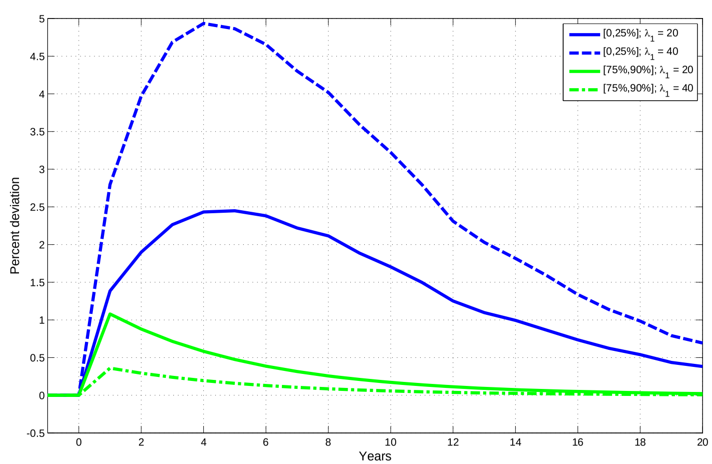
(a) Equity



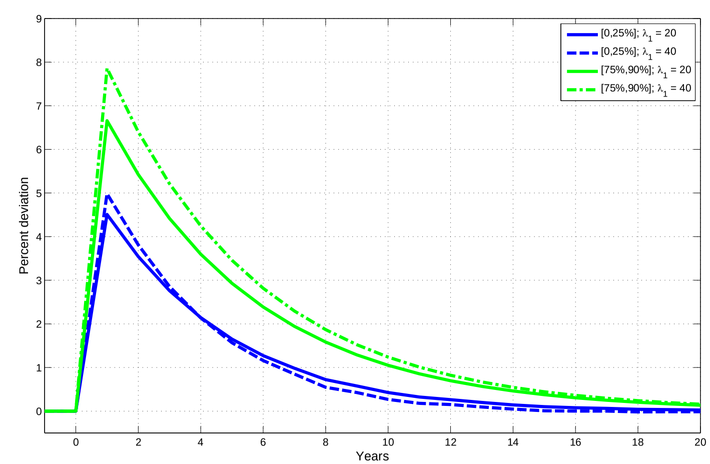
(b) Dividends



(c) Net worth



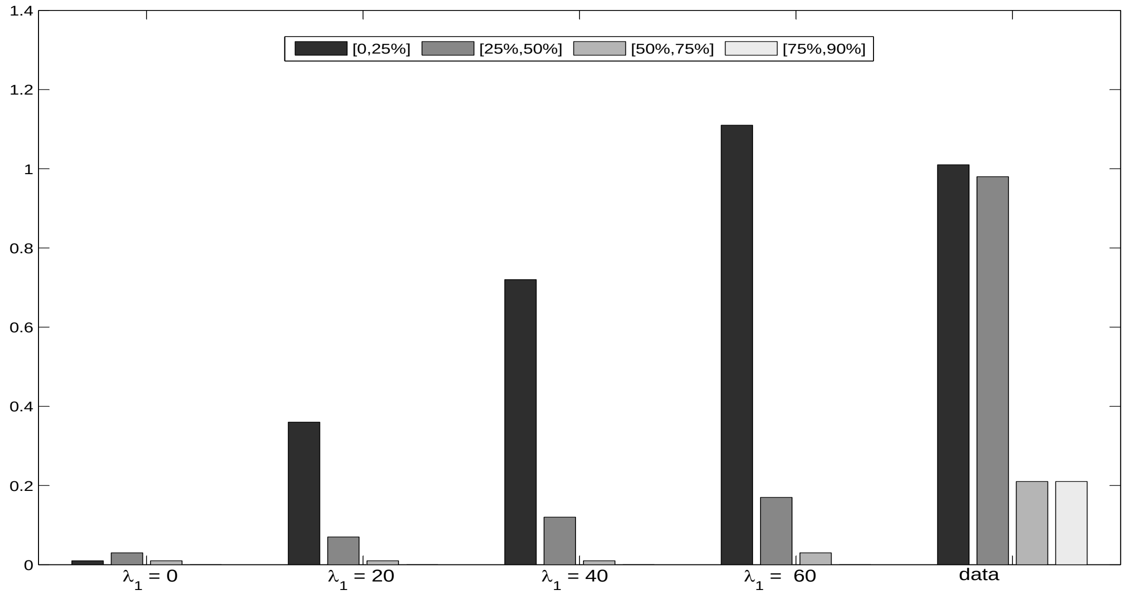
(d) Debt



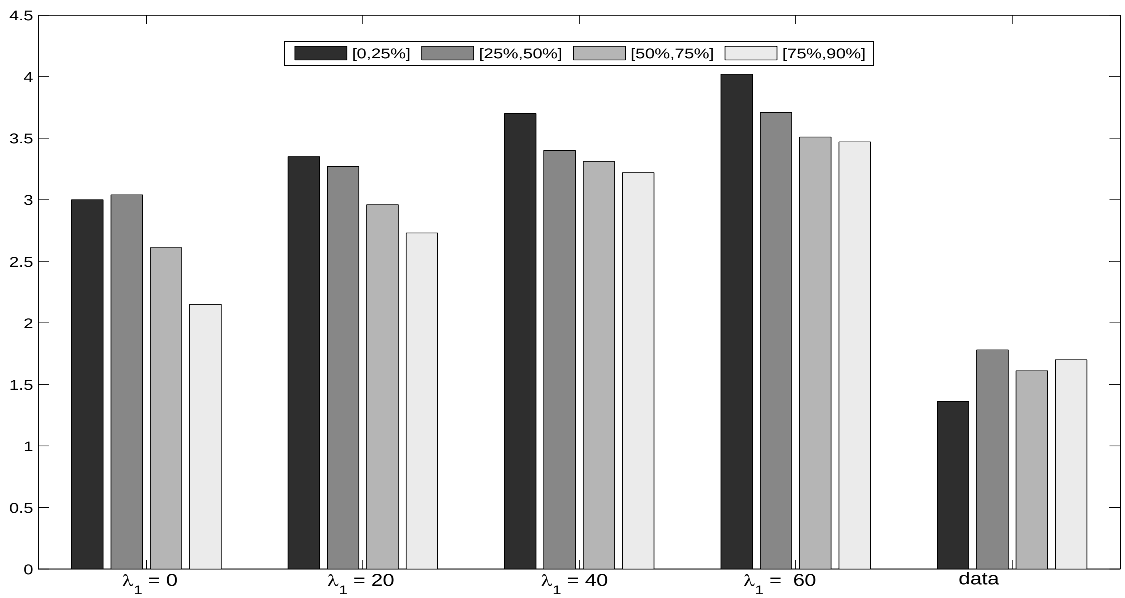
Notes: Panels plot the responses of the indicated variable to a positive one-standard-deviation aggregate productivity shock. The higher the value of λ_1, the more countercyclical equity issuance costs are.

Figure 2: Magnitudes of the debt and equity response to a productivity shock

(a) Equity



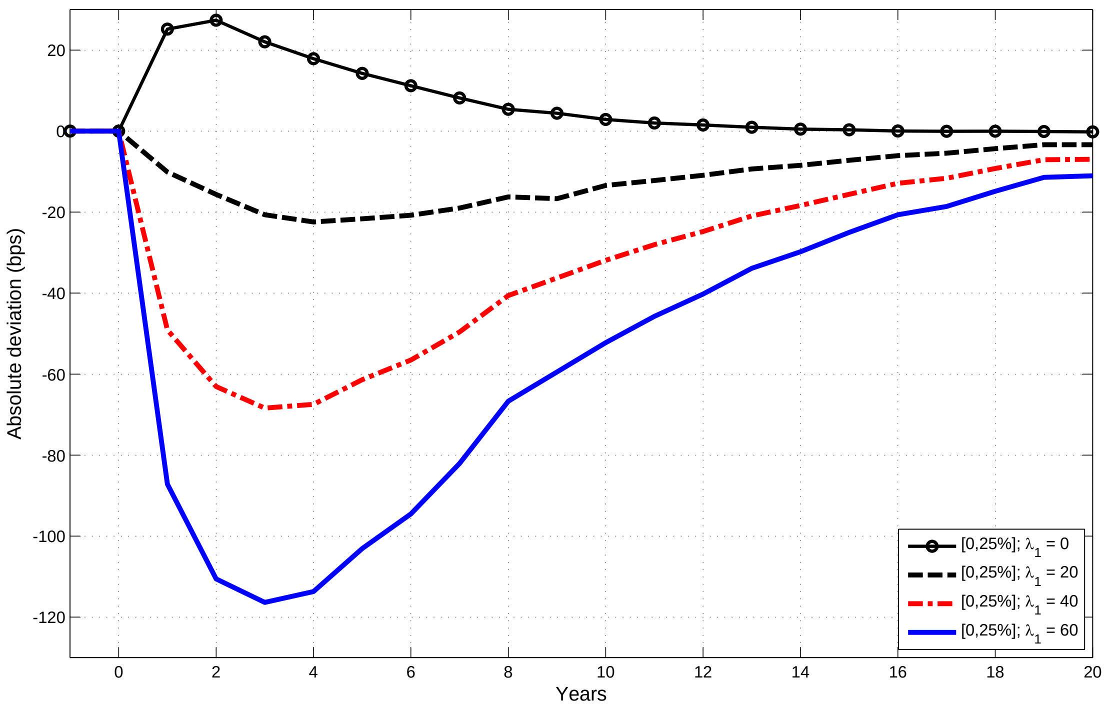
(b) Debt



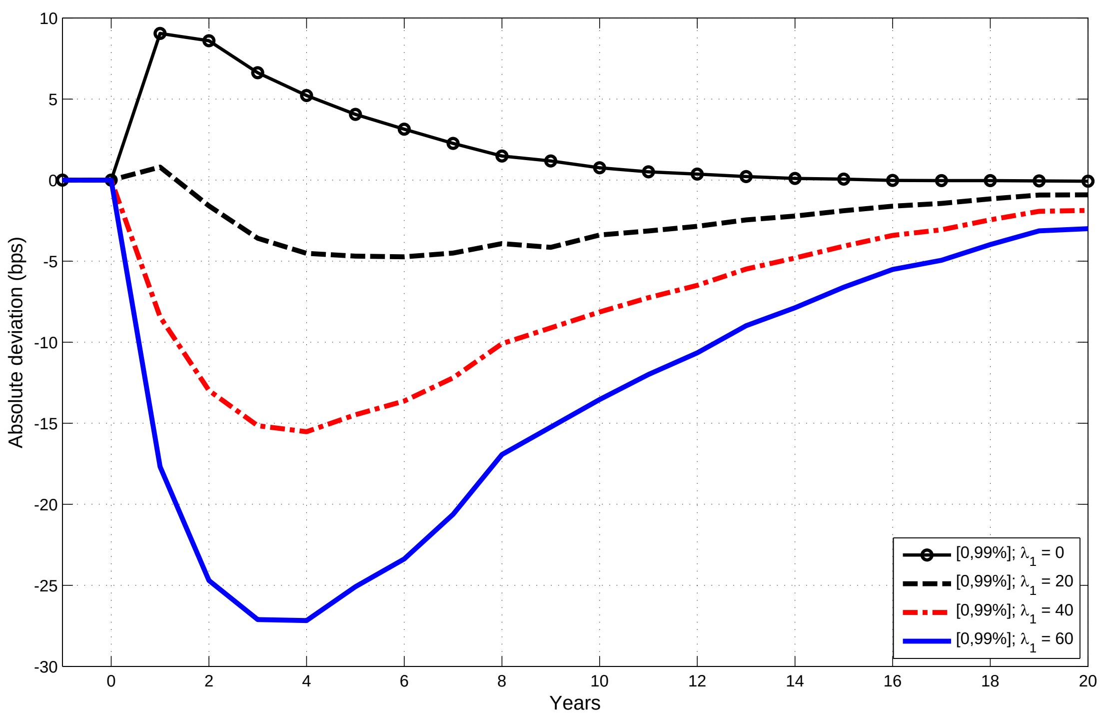
Notes: Panels compare the estimated panel coefficients from Covas and Den Haan (2010a) with the model response in the first period (year) to a positive one-standard-deviation aggregate productivity shock. The higher the value of λ_1, the more countercyclical equity issuance costs are.

Figure 3: Default rate responses to a productivity shock

(a) Small firms



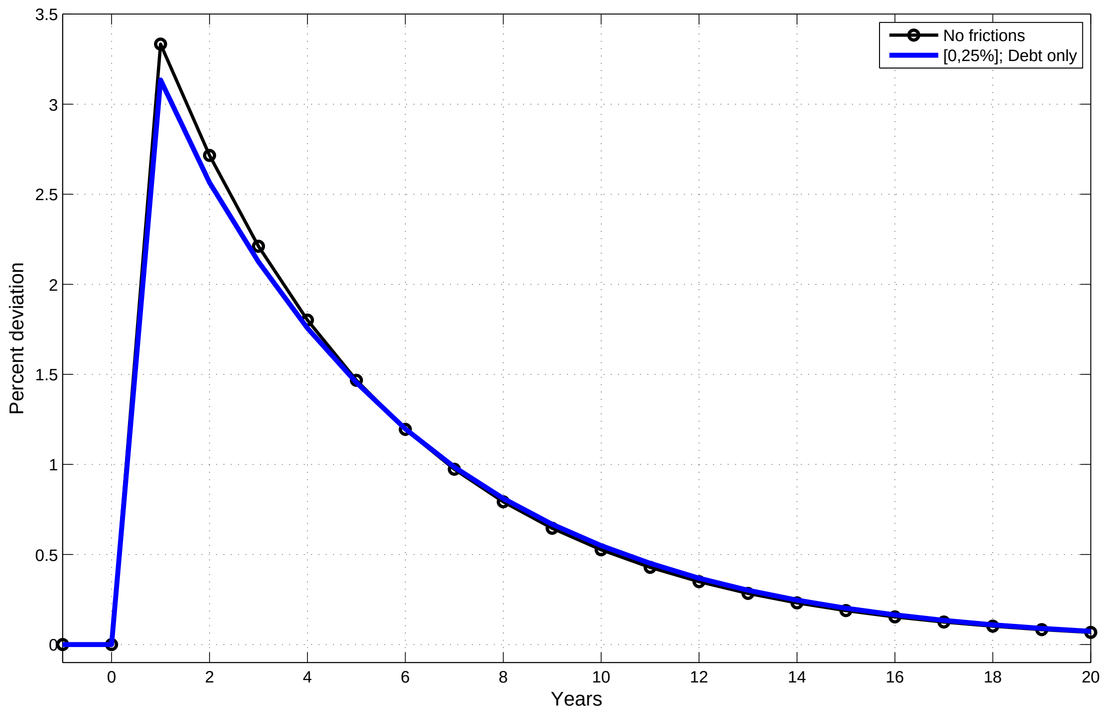
(b) All firms



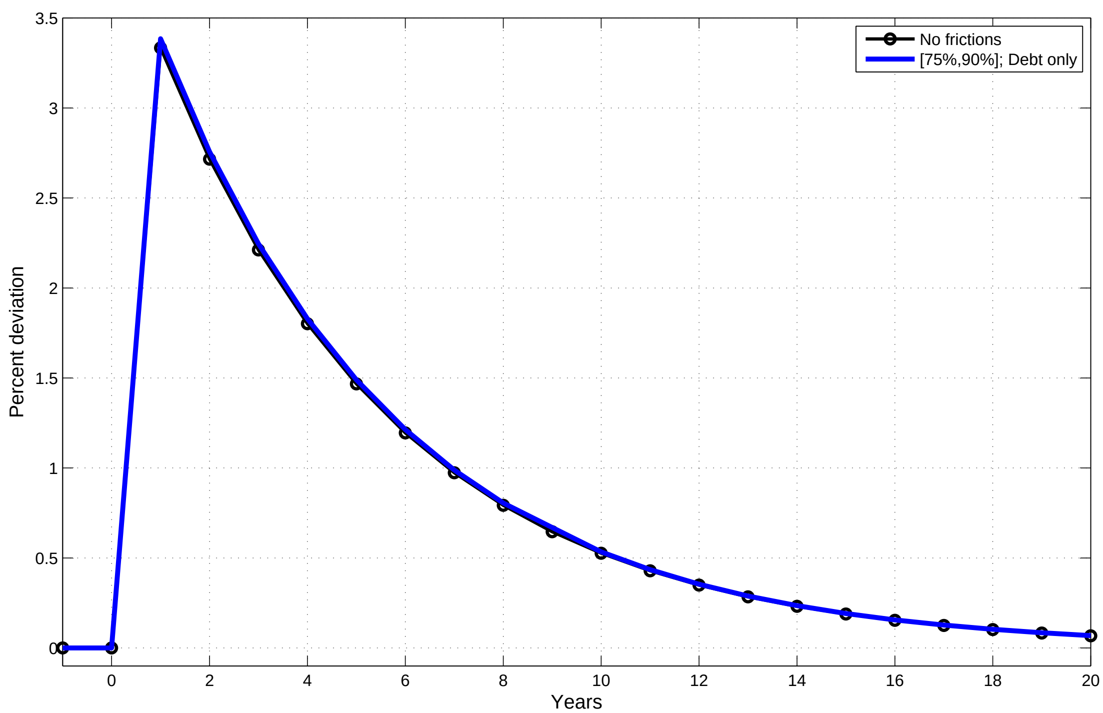
Notes: Panels plot the responses of the indicated variable to a positive one-standard-deviation aggregate productivity shock. The higher the value of λ_1, the more countercyclical equity issuance costs are.

Figure 4: Output responses to a productivity shock in the model with only debt

(a) Small firms



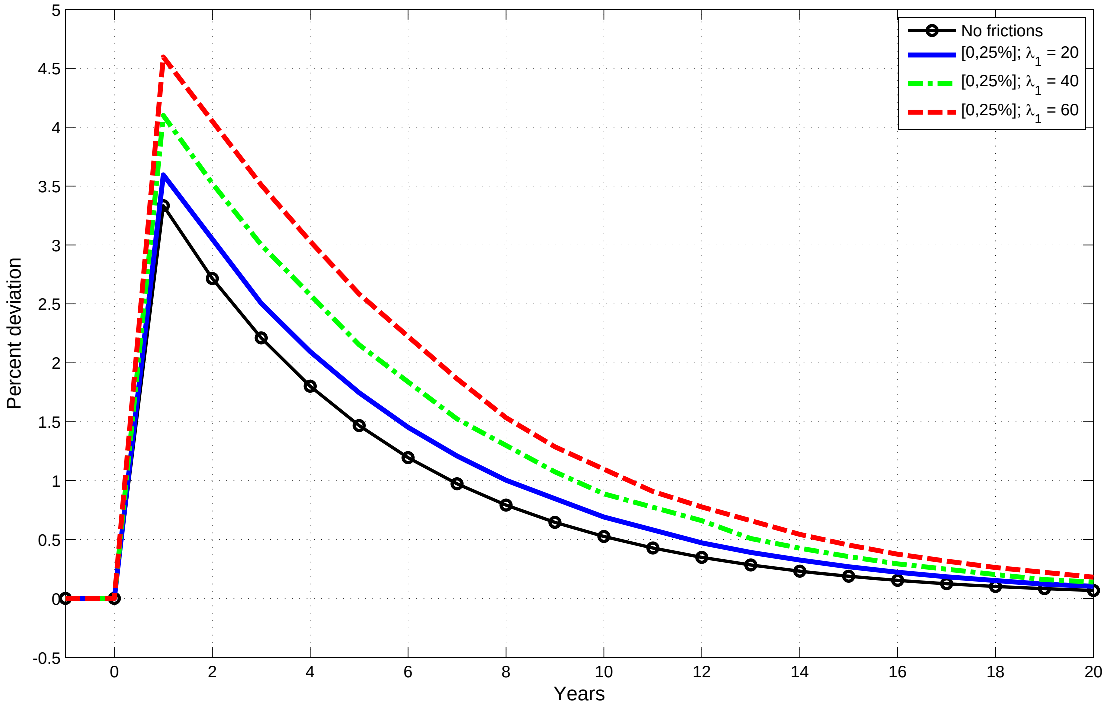
(b) Large firms



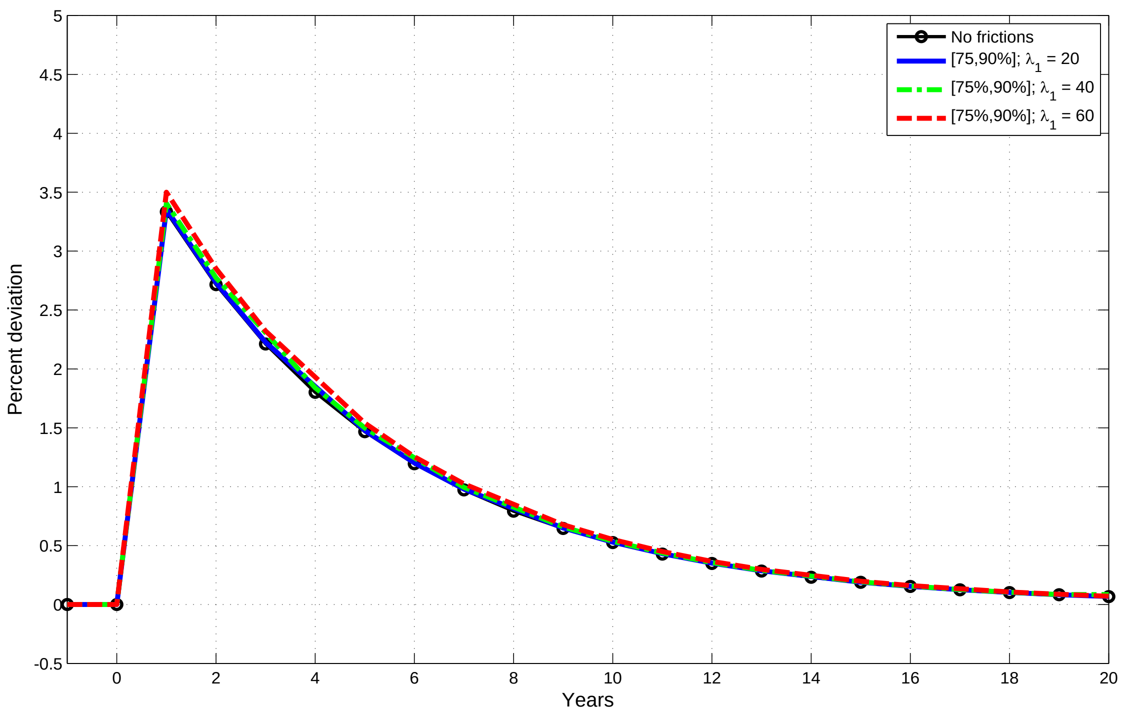
Notes: Panels plot the responses of the indicated variable to a positive one-standard-deviation aggregate productivity shock. In the "debt only" model, firms only have access to debt finance and the debt contract is subject to the standard bankruptcy friction.

Figure 5: Output responses to a productivity shock in the model with debt and equity

(a) Small firms

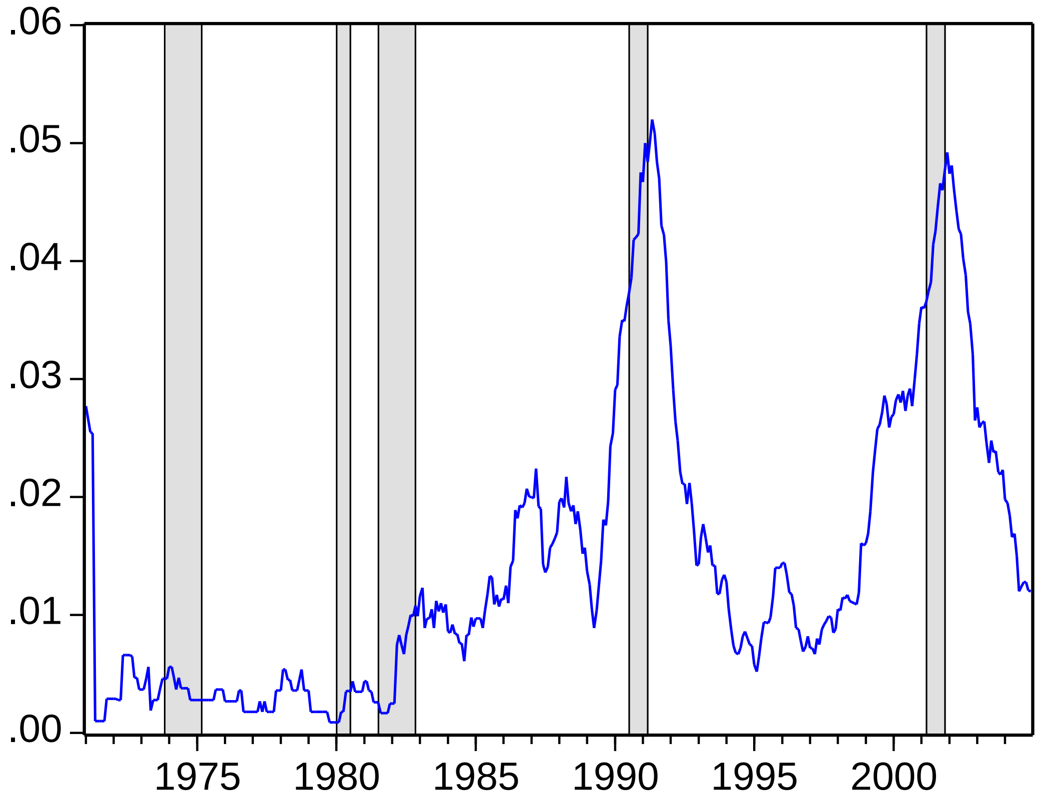


(b) Large firms



Notes: Panels plot the responses of the indicated variable to a positive one-standard-deviation aggregate productivity shock. The higher the value of λ_1, the more countercyclical equity issuance costs are.

Figure 6: Default Rate on Corporate Bonds



Notes: The default rate series is from Moody's (mnemonic USMDDAIW in Datastream) and it is for all corporate bonds in the US. The plot shows the annual default rate, i.e., the number of defaults during a year divided by the number of outstanding issuers at the beginning of the year, adjusted by the number of rating withdrawals during the year.

Table 1: Summary statistics

	size classes			
	[0, 25%]	[25%, 50%]	[50%, 75%]	[75%, 90%]
# of firms	795	769	772	473
% assets	0.5	1.7	5.9	13.3
$\Delta E/\Delta A$	0.880	0.568	0.346	0.223
$\Delta L/\Delta A$	0.275	0.409	0.515	0.605
$\Delta RE/\Delta A$	-0.156	0.021	0.144	0.185
$\Delta N/N$	27.2	12.1	8.2	5.2
$\Delta A/A$	26.1	15.0	11.3	9.0

	size classes			
	[90%, 95%]	[95%, 99%]	[99%, 100%]	all firms
# of firms	155	132	32	3128
% assets	12.7	32.1	33.8	100.0
$\Delta E/\Delta A$	0.183	0.128	0.134	0.211
$\Delta L/\Delta A$	0.659	0.661	0.638	0.616
$\Delta RE/\Delta A$	0.174	0.225	0.243	0.186
$\Delta N/N$	3.2	1.5	0.0	3.5
$\Delta A/A$	7.7	6.7	4.1	6.9

Notes: A equals the book value of assets; ΔE equals the change in stockholders' equity, which excludes accumulated retained earnings; ΔL equals the change in the book value of total liabilities; ΔRE is the change in the balance-sheet item for retained earnings; $\Delta N/N$ equals the percentage change in the number of workers. See Appendix A and Covas and Den Haan (2010a) for details on the data series used.

Table 2: Cyclical behavior of debt and equity - correlation

	data	model			
		$\lambda_1 = 0$	$\lambda_1 = 20$	$\lambda_1 = 40$	$\lambda_1 = 60$
debt and GDP					
$[0, 25\%]$	0.85	0.99	0.99	0.99	0.98
$[25\%, 50\%]$	0.87	0.99	0.99	0.98	0.98
$[50\%, 75\%]$	0.83	0.98	0.98	0.98	0.98
$[75\%, 90\%]$	0.68	0.98	0.98	0.99	0.99
$[90\%, 95\%]$	0.67	0.98	0.98	0.99	0.99
$[95\%, 99\%]$	0.29	0.98	0.99	0.99	0.99
$[99\%, 100\%]$	-0.01	0.98	0.99	0.99	0.99
$[0, 99\%]$	0.63	0.99	0.99	0.99	0.99
$[0, 100\%]$	0.37	0.99	0.99	0.99	0.99
equity and GDP					
$[0, 25\%]$	0.53	0.17	0.45	0.47	0.48
$[25\%, 50\%]$	0.57	0.76	0.54	0.48	0.46
$[50\%, 75\%]$	0.43	0.86	0.76	0.62	0.56
$[75\%, 90\%]$	0.41	0.01	0.01	0.01	0.01
$[90\%, 95\%]$	0.30	0.01	0.01	0.01	0.01
$[95\%, 99\%]$	0.13	-	-	-	-
$[99\%, 100\%]$	-0.36	-	-	-	-
$[0, 99\%]$	0.35	0.49	0.48	0.48	0.48
$[0, 100\%]$	0.17	0.49	0.48	0.48	0.48

Notes: This table reports the correlation coefficients of HP-filtered equity issuance and corporate GDP. λ_1 indicates the extent to which the friction to raise equity is reduced when aggregate conditions improve. This friction is acyclical when $\lambda_1 = 0$ A hyphen indicates that firms in this firm group never issue any equity.

Table 3: Magnitudes of the debt and equity response to a productivity shock

	[0, 25%]	[25% 50%]	[50% 75%]	[75% 90%]	[90% 95%]	[95% 99%]	[99% 100%]
debt	1.36	1.78	1.61	1.70	2.45	1.44	2.44
equity	1.01	0.98	0.21	0.21	0.41	0.07	-0.01

Notes: This table displays the panel coefficients of Covas and Den Haan (2010a) that reflect with how much $\Delta D/A$ and $\Delta E/A$ increase when GDP increases with one standard deviation.

Table 4: Calibration

Parameter		Source		
β	1.022^{-1}	Zhang (2005)		
α	0.70	Cooper and Ejarque (2003)		
τ	0.296	Graham (2000)		
ρ	0.95^{4}	Cooley and Hansen (1995)		
Parameter		**Moment**	**Data**	**Model**
σ_ϵ	0.010	Volatility of asset growth	0.039	0.035
σ_ω	0.310	Default premium	119bp	106bp
δ_0	0.082	Investment to assets	0.133	0.134
δ_1	-2.72	Leverage	0.587	0.526
η	0.098	Fraction of dividend payers	0.469	0.394
μ	0.150	Default rate	0.022	0.020
λ_0	0.75	Equity issuance costs	0.057	0.056
λ_1	20	Vol. of equity iss. costs	0.007	0.010

Notes: The parameter β is the discount factor, α the curvature of technology, τ the tax rate, and ρ the persistence of the aggregate shock. The parameter σ_ϵ is the standard deviation of the aggregate shock, σ_ω the standard deviation of the idiosyncratic shock, δ_0 the depreciation rate, and δ_1 the stochastic depreciation parameter. The parameter η is the fixed cost, μ is the bankruptcy cost, and λ_0 the direct costs of equity issuance. Finally, λ_1 controls the volatility of equity issuance costs. The moments in the model are obtained by simulating an economy with 5,000 firms for 5,000 periods and discarding the first 500 observations. Asset growth is the growth rate of the book value of assets. The default premium is the estimated default spread on corporate bonds taken from Longstaff, Mithal, and Neis (2005). Investment includes capital expenditures, advertising, research and development, and acquisitions. Leverage equals liabilities divided by the book value of assets. Dividends is dividends per share by ex-date multiplied by the number of common shares outstanding. The default rate is the average of annual default rates for all corporate bonds. The sample period is from 1971 until 2004, except for the default rate series, which is from the period between 1986 and 2004. See Covas and Den Haan (2010a) for details on the data series used.