Inefficient continuation decisions, job creation costs, and the cost of business cycles

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Abstract

This paper develops a model according to which the costs of business cycles are nontrivial because they reduce the average level of output. The reason is an *interaction* between job creation costs and an agency problem. The agency problem triggers separations during economic downturns even though both the employer and the worker would be better off if the job was not discontinued, that is, affected jobs have strictly positive surplus values. Similarly, booms make it possible for more jobs to overcome the agency problem. These effects do not offset each other, because business cycles reduce the expected job duration for these jobs. With positive job creation costs, business cycles then reduce the creation of valuable jobs and lower average activity levels. Considering a wide range of parameter values, we find estimates for the cost of business cycles ranging from 2.03% to 12.7% of GDP.

JEL Classification: E24, E32

Key Words: Agency problem, welfare, permanent job loss

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1 Introduction

This paper documents that even modest business cycles can be quite costly in a very simple framework with risk neutral agents and the following four features. First, creating a job requires a fixed job creation cost. Second, there is job heterogeneity. In particular, jobs differ in their productivity level and creation cost. Third, the employer and employee face an agency problem when deciding to continue an existing job and similarly when deciding to start operating a newly created job. This leads to job separations that are inefficient in the sense that the joint benefits for the employer and the employee when the job is discontinued are lower than what would be earned if the job could continue. Fourth, the agency problem is such that the number of inefficient job separations increases during a recession.

In this paper, we generate the last two features using the contractual fragility framework of Ramey and Watson (1997), in which participants cannot commit to putting in high effort. But these two features are quite typical outcomes in business cycle models with an agency problem.[1] The agency problem introduces an effort constraint into the model; if total revenues are not high enough, then the effort constraint is not satisfied. If there are enough resources available, then contracts can be written such that it is optimal for both participants to stay in the existing relationship *and* to put in high effort. If there are not enough resources available, then no such contracts can be written *even* though joint revenues generated under high effort (net of the cost of effort) exceed the joint revenues generated outside the relationship and also exceed the joint revenues generated under low effort. Consequently, potential jobs with revenues that are too low are not created and existing jobs for which revenues drop below the required level are discontinued.

A job's productivity level is affected by a stochastic aggregate variable and by a job-specific variable. To study the impact of business cycles, we analyze the effect of introducing mean-preserving fluctuations in the aggregate productivity level. The four features mentioned above do not provide a reason for business cycles to be costly when considered separately. But the interaction between them does make business cycles costly. In particular, we show that fluctuations are costly because they deter job creation and lower the *average* level of output produced. The following paragraph describes why this happens.

Some jobs that do not satisfy the effort constraint in a world without business cycles could do so during a boom. Business cycles are beneficial for these jobs. There are also jobs that always satisfy the effort constraint when there are no business cycles but would no longer be able to do so during a recession. Business cycles are costly for these jobs. The job destruction during a recession that is induced by the agency problem increases the cost of recessions as in Caballero and Hammour

[1] In appendix A2 of Den Haan and Sedlacek (2009), we show that the same type of result can be generated in a model in which there is an agency problem in obtaining firm finance.

(2005), but the job creation during a boom that is made possible by a relaxation of the agency problem increases the benefits of booms. If there are no job creation costs, then there is no robust reason why the negative effects during a recession would significantly outweigh the positive effects during a boom. With job creation costs, however, it is no longer true that the benefits of extra jobs during a boom should roughly offset the losses of less jobs during a recession. In a world without business cycles, all jobs in our model survive until they are hit by an exogenous destruction shock. In contrast, in a world with business cycles the jobs that only satisfy the effort constraint during a boom only last until the next recession. That is, the agency problem is such that one cannot compensate an inability to satisfy the effort constraint during bad times with slack during good times. Consequently, business cycles necessarily reduce the expected duration of affected jobs. This means that the positive and the negative effects of business cycles do not offset each other, not even if the output gains of the jobs temporarily made possible by the boom offset the output losses of the jobs made temporarily impossible by the recession. There are two reasons. First, jobs created during booms and eliminated during the subsequent downturn have to pay job creation costs more often, since their expected job duration has gone down. Second and more importantly, some jobs are no longer created, since it is too costly to create them given that their expected duration has been shortened by business cycles. These jobs may be marginal in terms of being able to satisfy the effort constraint of the agency problem, but they have a strictly positive surplus from a social welfare point of view. Consequently, their disappearance has non-trivial welfare consequences.

In terms of magnitude, business cycles are as bad as a permanent drop in output of several percentage points. In contrast, the classic Lucas (1987) paper reports an estimate for the cost of business cycles that is less than one tenth of a percentage point of consumption when the coefficient of risk aversion is equal to 10. Our model does not rely on high risk aversion to explain why moderate fluctuations like business cycles are costly. In fact, we assume that agents are risk neutral. As pointed out in Lucas (2003), if agents are highly risk averse, then the question arises why high risk aversion does not show up in, for example, the diversification of individual portfolios, the level of insurance deductibles, or the wage premiums of jobs with high earnings risk.

Our paper fits into a line of research that investigates the effect of uncertainty on the level or growth rate of output, which Lucas (2003) refers to as *"... a promising frontier on which there is much to be done"*.[2] Besides the assumption of linear utility, our framework differs from related papers in that we focus on different model features to generate the relationship between volatility and the level of real activity. Those are costly job creation and an agency problem affecting the ability to make efficient

[2]In section 6, we discuss the related theoretical literature and papers that provide empirical support for the view that business cycles do not leave the long-run growth path unchanged.

decisions, which are two features often found in business cycle models.

The rest of this paper is organized as follows. In section 2, we develop our framework. In section 3, we discuss model properties. In particular, we discuss what types of jobs are affected by business cycles and why. We also show that some jobs are affected by *arbitrarily small* business cycles. In section 4, we discuss the calibration and our procedure to calculate the cost of business cycles. The results are presented in section 5. The literature is discussed in section 6 and the last section concludes.

2 Model

In this section, we present our model that is characterized by job heterogeneity, job creation costs, and an agency problem affecting the ability to implement the first-best outcome, i.e., the agency problem prevents some choices that are valuable from a social welfare point of view. The particular agency problem is the contractual fragility problem of Ramey and Watson (1997) that models the collaboration between an employer and an employee.

Agents and agents' characteristics. There are two types of agents: workers and entrepreneurs. We assume that agents are risk neutral to accentuate that business cycles can be costly *even* when agents are risk neutral. Workers are indexed by i_w and are characterized by productivity $\phi_p(i_w)$. For each productivity level there is a continuum of workers and also a continuum of entrepreneurs. Entrepreneurs are indexed by i_e and are characterized by the amount they have to pay to create a job, $\phi_c(i_e)$. We abstract from matching frictions and assume that for each worker with productivity $\phi_p(i_w)$ there is an entrepreneur that could create the job.[3] That is, if the entrepreneur decides to create a job, then a relationship can be established instantaneously. This environment can be simply described as one with a continuum of types of jobs indexed by i, where each job is characterized by a productivity level, $\phi_p(i)$, and a job creation cost, $\phi_c(i)$.

The values of $\phi_p(i)$ and $\phi_c(i)$ are assumed to be constant through time. That is, all idiosyncratic uncertainty is resolved at the beginning of time. In section 7, we clarify why our results remain valid if these characteristics would be time varying. The joint distribution of ϕ_c and ϕ_p has continuous support without point mass. The density is denoted by $f\left(\phi_c, \phi_p\right)$. Each period, an individual firm could be hit by an exogenous shock that will lead to job destruction. This shock occurs with

[3]Key in our analysis is the question how business cycles affect the viability of jobs given their productivity level, $\phi_p(i_w)$, and their startup cost, $\phi_c(i_e)$. With matching frictions, it takes on average more than one vacancy to create a job. For the issues addressed in this paper, this would be similar to an increase in $\phi_c(i_e)$.

probability $1-\rho$. The job can be recreated by paying the job creation costs again. Production of an active job i, $y_t(i)$, is given by

$$y_t(i) = \phi_p(i)\Phi_{p,t}, \tag{1}$$

where $\Phi_{p,t}$ is aggregate productivity. An unemployed worker receives $\mu(i)$ in the form of home production and/or leisure. For now, we assume that $\mu(i)$ is constant, which simplifies the exposition, but later we will assume that $\mu(i)$ is proportional to $\phi_p(i)$, which simplifies the calculations.[4] We do not include transfers in the analysis. Unemployment benefits create a wedge between the private and social benefits of not working. We want to make clear that our mechanism does not depend on the presence of such a wedge.

From now on, we suppress the i index, but the reader should keep in mind that ϕ_p, ϕ_c, and μ are the only exogenous variables that can vary across jobs.

Aggregate fluctuations. Two different assumptions about $\Phi_{p,t}$ are considered. Under the first assumption, $\Phi_{p,t}$ is constant through time and equal to 1. In this case, jobs are heterogeneous, but face an unchanging macroeconomic environment. Under the second assumption, $\Phi_{p,t}$ is a stochastic variable that varies across time according to the law of motion specified in Krusell and Smith (1998). In particular, $\Phi_{p,t}$ can take on two values, Φ_+ in a boom and Φ_- in a recession. The probability of transitioning out of a boom, $1-\pi$, is equal to the probability of transitioning out of a recession. This implies that the expected durations of staying in a boom and a recession are equal to each other. Moreover, $\Phi_+ - 1 = 1 - \Phi_- = \Delta_{\Phi_p}$, which ensures that $\mathrm{E}[\Phi_{p,t}] = 1$.

Contractual fragility framework of Ramey and Watson (1997). Two types of decisions are made by entrepreneurs and workers. First, entrepreneurs have to decide whether to pay ϕ_c and to *create* jobs. Second, members in an ongoing relationship have to choose an effort level and decide whether they want to *continue* the relationship. This decision is affected by contractual fragility as modelled in Ramey and Watson (1997). We start with the second decision.

In the agency problem of Ramey and Watson (1997), employers and employees have to decide whether they put in high or low effort. We assume that only the entrepreneur faces such an effort choice, which simplifies the exposition, but does not affect the analysis. At the beginning of the period, the worker has to decide whether he stays in the relationship or whether he quits. If the worker quits, then the job is destroyed. When making decisions, agents take into account both current-period and future benefits. The analysis is simplified by the following assumptions: (i) no

[4] Since the value of μ only matters for jobs with a similar value for ϕ_p, it doesn't matter much which choice is made for the results.

Table 1: Current-period payoffs for worker and entrepreneur

		entrepreneur	
		high effort	low effort
worker	stay	$\left(w_t, \phi_p \Phi_{p,t} - w_t\right)$	$\left(\phi_\chi \phi_p \Phi_{p,t} - \chi_e - \chi_w, \chi_e\right)$
	quit	$(\mu, 0)$	$(\mu, 0)$

Notes: If the entrepreneur puts in low effort, then she receives χ_e instead of the agreed upon $\phi_p \Phi_{p,t} - w_t$, output is reduced by a factor ϕ_χ, and the worker suffers (additional) disutility of χ_w.

matching frictions and (ii) bad behavior by the entrepreneur severs the relationship, but not the job.[5,6]

Under these assumptions, we can focus on current-period payoffs, which are given in table 1. If the entrepreneur chooses high effort, then firm revenues are equal to $\phi_p \Phi_{p,t}$, the worker receives the agreed upon wage w_t, and the entrepreneur receives the residual, $\phi_p \Phi_{p,t} - w_t$. Alternatively, the entrepreneur can put in low effort. "Putting in low effort" should be interpreted broadly. It could mean diversion of funds, but it could also mean, for example, exploiting the worker or deviating from the original business plan by taking on additional risk. If the entrepreneur puts in low effort, then firm revenues are equal to $\phi_\chi \phi_p \Phi_{p,t}$ with $\phi_\chi < 1$, the entrepreneur's payoff equals χ_e, and the worker's net payoff is equal to $\phi_\chi \phi_p \Phi_{p,t} - \chi_e - \chi_w$ with $\chi_w \geq 0$, where χ_w is the disutility imposed on the worker by the "low effort" choice of the entrepreneur.[7] Note that

$$\phi_p \Phi_{p,t} > \phi_\chi \phi_p \Phi_{p,t} - \chi_w. \tag{2}$$

That is, putting in low effort is harmful when *joint* benefits are considered. Nevertheless, it may be optimal for the entrepreneur to put in low effort. If the worker does not quit, then the entrepreneur is only willing to put in high effort if

$$\phi_p \Phi_{p,t} - w_t \geq \chi_e, \tag{3}$$

[5]If shirking by the entrepreneur also destroys the job, then the net-benefit of shirking for the entrepreneur would be time-varying, since it would include losing the value of the job. This would complicate the expressions, but not the underlying idea that overcoming the agency problem is easier for higher values of $\Phi_{p,t}$.

[6]The assumption that shirking severs the relationship means that it is evident that a worker will never work for less than μ, not even if wage payments below μ are possibly offset by higher payments in the future.

[7]If bad behavior by the entrepreneur would destroy the job, then this cost would be part of the net-benefit χ_e.

where we use the assumption that the entrepreneur's choice to put in low effort does not affect her continuation value.

The worker is only willing to stay in a job if

$$w_t \geq \mu. \tag{4}$$

A necessary and sufficient condition to satisfy both the incentive compatibility condition (3) and the worker participation condition (4) is given by

$$\phi_p \Phi_{p,t} \geq \chi_e + \mu. \tag{5}$$

If we let $\chi = \chi_e + \mu$, then we can write this condition as

$$\phi_p \Phi_{p,t} \geq \chi. \tag{6}$$

Consistent with the terminology in Ramey and Watson (1997), we refer to the requirement given in equation (6) as the *effort constraint*. If a job does not satisfy this constraint, then it is not possible for the worker to get at least his outside option. This is known to the worker, since all relevant information is known at the beginning of the period. Consequently, a worker would not take such a job. Moreover, he would quit if he is in a job where $\phi_p \Phi_{p,t}$ falls below χ.

The jobs for which the agency problem matters. The interesting jobs are those with a value for ϕ_p such that

$$\mu < \phi_p \Phi_{p,t} < \chi_e + \mu = \chi.$$

If $\phi_p \Phi_{p,t} < \mu$, then the job does not satisfy the effort constraint, but it would not operate in the first-best solution either. The first-best solution is also implemented if $\phi_p \Phi_{p,t} > \chi$, because jobs with this level of revenues satisfy the effort constraint. If $\mu < \phi_p \Phi_{p,t} < \chi$, then the job would produce in the first-best solution, but not when the agents face the agency problem. For these values of $\phi_p \Phi_{p,t}$, it is not possible to both pay the entrepreneur enough so that she will not put in low effort and pay the worker enough so that his wage exceeds μ. In this case, the job is not viable. The idea of the contractual fragility of Ramey and Watson (1997) is that no verifiable contract can be written that will prevent the entrepreneur from putting in low effort.[8] The entrepreneur may promise that she will pay the worker a wage above μ and that she will put in high effort, but if $\phi_p \Phi_{p,t} < \chi_e + \mu$, then the entrepreneur cannot both pay the worker more than μ and satisfy her own incentive compatibility condition. The worker knows that the entrepreneur will face this dilemma and will go for his outside option and earn μ.

[8] For example, even when it is "observed" that the entrepreneur puts in low effort, this would not provide any evidence that could be used in court.

The cut-off level for ϕ_p. Let $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{p,t})$ be the value of ϕ_p such that equation (6) holds with equality when there are business cycles, that is, when $\Delta_{\Phi_p} > 0$. Thus,

$$\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{p,t})\,\Phi_{p,t} = \chi \text{ or } \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{p,t}) = \frac{\chi}{\Phi_{p,t}}.$$

Consequently, the fraction of jobs that do not satisfy the effort constraint decreases in a boom and increases in a recession. If χ is not constant, then $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{p,t})$ remains countercyclical as long as the effect of $\Phi_{p,t}$ on regular market production is stronger than the effect on the benefits associated with low effort.[9]

Job creation decision and wage setting. Jobs can be created by paying a start-up cost, ϕ_c. When a job has been inactive, then ϕ_c would have to be paid once more to restart it. That is, one cannot simply mothball a job and restart it as if there had been no interruption. One possible reason for this is that it may take some time and effort before the job is operating at its potential productivity of ϕ_p again.

In our framework, the entrepreneur pays the job creation costs and compares these costs with the Net Discounted Value (NPV) of the revenues she receives. Entry occurs whenever

$$N_{e,\mathrm{bc}}(\phi_c, \phi_p, 1, \Phi_{p,t}) - \phi_c \geq \beta \mathrm{E}_t \left[N_{e,\mathrm{bc}}(\phi_c, \phi_p, 0, \Phi_{p,t+1}) \right],$$

where $N_{e,\mathrm{bc}}(\phi_c, \phi_p, 1, \Phi_{p,t})$ is the discounted value of the job's current and future earnings accruing to the entrepreneur when the job creation costs have been paid, and $N_{e,\mathrm{bc}}(\phi_c, \phi_p, 0, \Phi_{p,t})$ is the discounted value of earnings when the job creation costs have not been paid.[10] The bc (no-bc) subscript indicates that the value refers to the case with (without) business cycles. When the job creation costs are not paid, then it remains possible to create the job at a future date. Job creation decisions are efficient if entry occurs whenever

$$N_{\mathrm{bc}}(\phi_c, \phi_p, 1, \Phi_{p,t}) - \phi_c \geq \mu + \beta \mathrm{E}_t \left[N_{\mathrm{bc}}(\phi_c, \phi_p, 0, \Phi_{p,t+1}) \right], \tag{7}$$

where the discounted values now pertain to *joint* earnings. We denote the cut-off level of ϕ_c by $\tilde{\phi}^*_{c,\mathrm{bc}}(\phi_p, \Phi_{p,t})$ when the job is affected by the agency problem and by $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_{p,t})$ when it is not. We use different notation, because the agency problem affects the cut-off level for ϕ_c as discussed in detail below.

[9]Interestingly, all that is needed for our story to work is that $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{p,t})$ is cyclical, either procyclical or countercyclical. The only case that has to be ruled out is that the benefits associated with low effort are proportional to $\Phi_{p,t}$. In that case, business cycles would not shorten the expected job duration.

[10]Precise definitions and derivations can be found in appendices A and B. $N_{\mathrm{e,bc}}(\phi_p, \phi_c, 0, \Phi_{p,t})$ is equal to $N_{\mathrm{e,bc}}(\phi_p, \phi_c, 1, \Phi_{p,t}) - \phi_c$ when entry is optimal in the current period and is equal to $\beta \mathrm{E}_t \left[N_{\mathrm{e,bc}}(\phi_p, \phi_c, 0, \Phi_{p,t+1}) \right]$ when entry is not optimal in the current period.

The entrepreneur receives a share ω_e of the surplus, $\phi_p \Phi_{p,t} - \mu$. We present results for two cases. In the first case, wages are such that the job creation decision is efficient. This would happen, for example, if $\omega_e = 1$ and the wage rate is equal to μ. Efficient job entry would not be jeopardized if wages exceed μ as long as the value of ϕ_p (ϕ_c) is sufficiently high (low). If the job creation decision is efficient, then changes in the wage rule correspond to transfers among (risk neutral) agents that do not affect our per capita welfare calculations. When $\omega_e = 1$, average job creation costs turn out to be implausibly high. The main reason to consider this case is to make clear that business cycles are costly even if the job creation decision is efficient. We also consider the case when the entrepreneur receives a smaller (and more realistic) share of the revenues. Then entry is no longer efficient for all jobs, and the average of job creation costs takes on plausible values.

Welfare loss measure. We measure the impact of business cycles on individual jobs as the permanent increase (or decrease) in per-period income that would make the entrepreneur and worker in a world *with* business cycles as well off as they would be in a world *without* business cycles. To standardize the measure we scale by market production, ϕ_p. The formula is given in the following definition.

Definition 1 *The impact of business cycles on an individual job is given by*

$$L(\phi_c, \phi_p, \Delta_{\Phi_p}) = (1-\beta)\left(\frac{N_{no\text{-}bc}(\phi_c, \phi_p, 0) - E\left[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})\right]}{\phi_p}\right),$$

where $N_{no\text{-}bc}(\phi_c, \phi_p, 0)$ is the discounted value of earnings in a world without business cycles when the job creation cost has not been paid and

$$E\left[N_{bc}(\phi_c, \phi_p, 0, \Phi_{p,t})\right] = \frac{\left(N_{bc}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) + N_{bc}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p})\right)}{2}.$$

The aggregate welfare measure is obtained by aggregating the individual losses relative to aggregate output, that is,

$$L\left(\Delta_{\Phi_p}\right) = \frac{\int\int L\left(\phi_c, \phi_p, \Delta_{\Phi_p}\right)\phi_p f\left(\phi_c, \phi_p\right) d\phi_c d\phi_p}{Y},$$

where Y is an output measure.[11]

Our welfare measure compares the benefits of a job that has not yet been created in a world with business cycles to the benefits of a job that has not yet been created in a world without business cycles. We do not compare the output levels. The motivation is the following. Below, we will see that output will be lower in a world

[11] The precise definition is given in appendix C.1.

with business cycles, because some jobs are not created in a world with business cycles. To assess the cost of business cycles properly, one should not just compare output levels, one should take into account the beneficial aspect that the economy with business cycles does not have to incur the cost of creating those jobs.

3 Model properties

In this section, we describe the qualitative features of the model. We start in section 3.1 with a graphical representation of the affected groups. In section 3.2, we discuss the impact of arbitrarily small aggregate fluctuations. In section 3.3, we discuss the results if aggregate fluctuations take on non-trivial amplitudes. In section 3.4, we discuss the case when the job entry decision is not efficient.

3.1 Graphical representation of affected jobs

We start by presenting the case without business cycles. Also, we describe the model when job creation is efficient to highlight that our mechanism does not rely on inefficient job creation. If $\Phi_{p,t}$ is constant, then a job either *always* satisfies the effort constraint or *never* satisfies it. That is, the cut-off level for job productivity, ϕ_p, is constant and is given by

$$\tilde{\phi}_{p,\text{no-bc}} = \chi. \tag{8}$$

Jobs with a value of ϕ_p high enough to overcome the contractual fragility problem will be created as long as the job creation costs are low enough. We denote the cut-off level for ϕ_c in a world without business cycles by $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$. It is the level of ϕ_c for which equation (7) holds with equality. This implies that

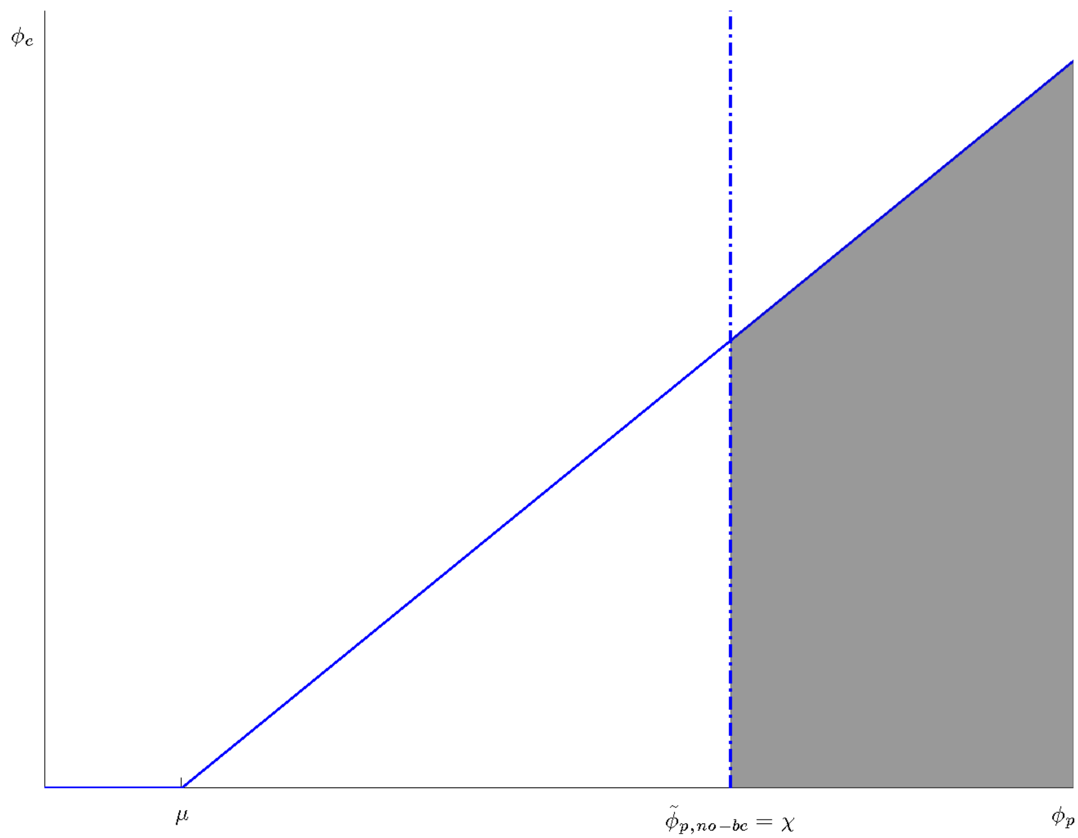
$$\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) = \frac{\phi_p - \mu}{1 - \beta\rho}. \tag{9}$$

The graphs in this section are based on the assumption that μ is the same for all jobs. Figure 1 displays the results when there are no business cycles. Jobs in the shaded area are created and produce market output, since their value of ϕ_p exceeds χ and their job creation costs are low enough. If the effort constraint is not satisfied, then job creation will not occur, no matter how low the job creation costs are.

Figure 2 shows the case with business cycles. Since agents are risk neutral, business cycles only affect agents' utility if aggregate fluctuations lead to different decisions. There are two types of jobs that are affected by business cycles.

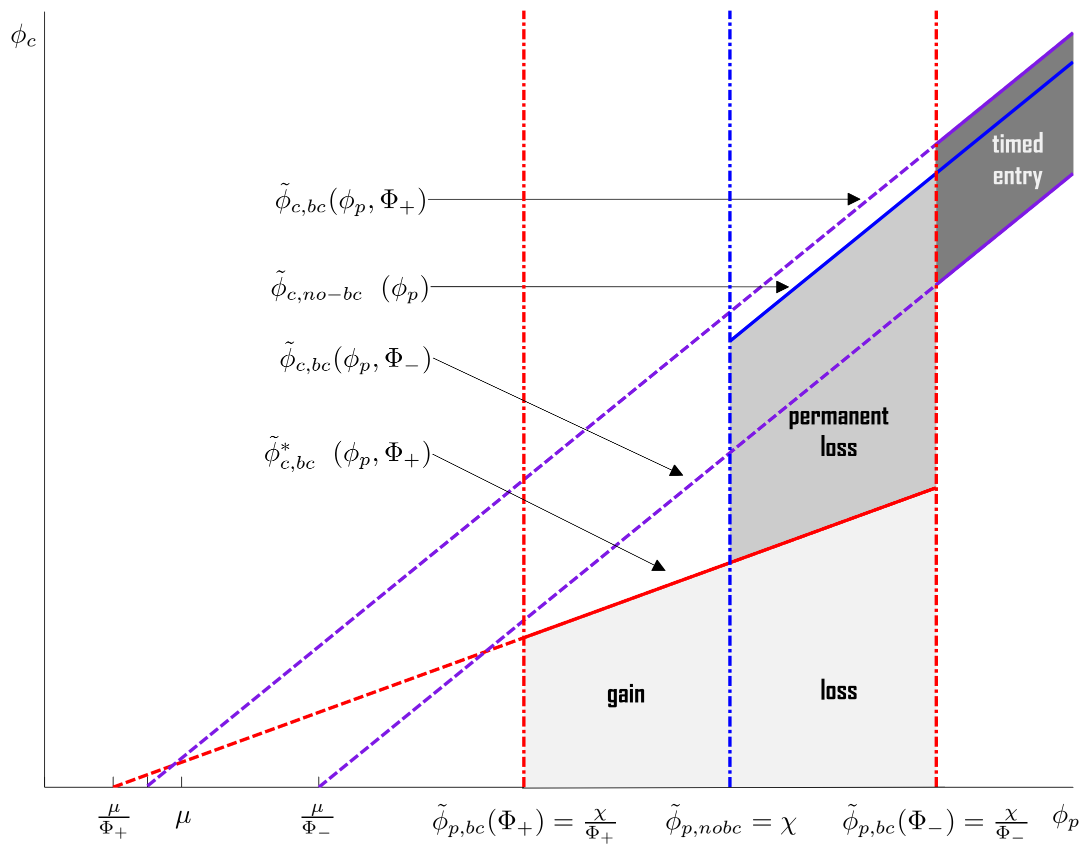
Timed-entry and fragile jobs. The first type of job that is affected by business cycles is a "timed-entry" job. Timed-entry jobs are jobs that (i) have a value of ϕ_p

Figure 1: Jobs operating in a world without business cycles



Notes: The shaded area in this graph indicates the jobs that (i) have a high enough value for job productivity, ϕ_p, to satisfy the effort constraint, $\phi_p \geq \chi$, and (ii) have a low enough value for job creation costs, ϕ_c, so that job creation is profitable. The graph is based on the assumption that μ is the same for all jobs.

Figure 2: Jobs affected by business cycles



Notes: ϕ_p is job productivity and ϕ_c is job creation cost. Φ_- (Φ_+) is the value of aggregate productivity in a recession (boom). $\tilde{\phi}_p$ is the lowest value of ϕ_p that satisfies the effort constraint. $\tilde{\phi}^*_c$ $\left(\tilde{\phi}_c\right)$ is the highest value of ϕ_c such that the entrepreneur wants to create the job if business cycles do (do not) shorten job duration because of the agency problem. The shaded areas in this graph indicate the jobs that are affected by business cycles. Light grey: Cyclical fragile jobs that satisfy the effort constraint during a boom and do not satisfy it during a recession in a world with business cycles. Moreover, job creation costs are low enough so that these jobs operate during booms. Jobs in the "gain" ("loss") area never (always) operate in a world without business cycles. Darker grey: Permanent-loss fragile jobs that satisfy the effort constraint during a boom, but their job creation costs are too high to make creating the job worthwhile given that the agency problem will force termination during a recession. Darkest grey: Timed-entry jobs. The graph is based on the assumption that μ is the same for all jobs.

that is such that they are never affected by the agency problem and (ii) have a value of ϕ_c such that $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$, that is, jobs are created during booms but not during recessions. Timed-entry jobs for which $\phi_c > \tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ ($\phi_c < \tilde{\phi}_{c,\text{no-bc}}(\phi_p)$) are never (always) created in a world without business cycles, whereas they are only created during booms in a world with business cycles. Business cycles only affect the job creation decision of timed-entry jobs. A timed-entry job that already has been created continues to operate—independent of the state of the aggregate economy—until it is hit by an exogenous destruction shock.

The second type of job affected by business cycles is a "fragile" job. These jobs have a value of ϕ_p such that the effort constraint is *not* satisfied in a recession, but is satisfied in a boom. These jobs are clearly affected by business cycles. Jobs with a value for ϕ_p such that $\tilde{\phi}_{p,\text{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\text{no-bc}}$ never satisfy the effort constraint in a world without business cycles, whereas they can (cannot) overcome the effort constraint during booms (recessions) in a world with business cycles. Jobs with a value for ϕ_p such that $\tilde{\phi}_{p,\text{no-bc}} \leq \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_-)$ are different in that they always satisfy the effort constraint in a world without business cycles, but are similar in that they can (cannot) overcome the effort constraint during booms (recessions) in a world with business cycles. To understand the consequences of business cycles, it is important to take into account job creation costs.

Fragile jobs and job creation costs. Consider two fragile jobs, both with zero job creation costs. One has a value of ϕ_p just above $\tilde{\phi}_{p,\text{no-bc}}$ and one has a value of ϕ_p just below $\tilde{\phi}_{p,\text{no-bc}}$. The job with a value of ϕ_p just below $\tilde{\phi}_{p,\text{no-bc}}$ gains from business cycles, because it can overcome the effort constraint during a boom and during that time period it earns a strictly positive surplus of $\phi_p - \mu$. By contrast, it can never satisfy the effort constraint in a world without business cycles. The job with a value of ϕ_p just above $\tilde{\phi}_{p,\text{no-bc}}$ is the mirror image. Business cycles are bad for this job, because the effort constraint is no longer satisfied all the time. For the pair there is a net loss, since the job that is temporarily made impossible by business cycles has a slightly higher value for ϕ_p than the job that is temporarily made possible. This effect of business cycles, due to affected jobs having different values of ϕ_p, turns out to be quantitatively not important.

Now consider fragile jobs with job creation costs that are positive, but low enough to ensure job creation even if job duration is shortened by business cycles. That is, $\phi_c \leq \tilde{\phi}^*_{c,\text{bc}}(\phi_p, \Phi_+)$. These jobs are referred to as cyclical fragile jobs, since they only operate during booms. Business cycles improve (worsen) welfare for these jobs when ϕ_p is just below (above) $\tilde{\phi}_{p,\text{no-bc}}$. In figure 2, these two types of fragile jobs are in the regions referred to as "gain" and "loss". In contrast to the case with zero job creation costs, the gains do not offset the losses when the job creation costs are positive *even* when the output differential between the two jobs is negligible. This will be discussed in section 3.2.

Consider fragile jobs with a value of ϕ_p above $\tilde{\phi}_{p,\text{no-bc}}$. Since these are fragile jobs, they stop operating whenever the economy enters a recession, which implies that their expected duration is shorter in the presence of business cycles. This means that business cycles reduce these jobs' cut-off values for ϕ_c. That is, $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right) < \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$. In other words, the reduction in the expected duration implies that for some jobs it is no longer worth it to pay the job creation costs even though it is worth doing so in a world without business cycles. This loss turns out to be the most important part of the cost of business cycles. These fragile jobs are in the area referred to as "permanent loss". In contrast to cyclical fragile jobs, which have a value of ϕ_c below $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$, there are only losses, namely for fragile jobs with a value of ϕ_p above $\tilde{\phi}_{p,\text{no-bc}}$. Jobs with a value of ϕ_p below $\tilde{\phi}_{p,\text{no-bc}}$ and a value of ϕ_c above $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$ are never created in a world without business cycles and neither in a world with business cycles.

3.2 The impact of arbitrarily small business cycles

The discussion in the last paragraph already makes clear that some jobs are better off and some jobs are worse off in the presence of business cycles. Here we analyze these gains and losses formally by considering arbitrarily small business cycles. By considering arbitrarily small business cycles, we can derive relatively simple analytical expressions with which we can compare the different outcomes, for example, the gains and losses of cyclical fragile jobs. The analysis based on arbitrarily small business cycles also highlights another feature of our setup, namely a discontinuity. That is, when the magnitude of business cycles approaches—but remains distinct from—zero, then some effects of business cycles approach zero, whereas some effects do *not* and in fact can remain quite large. This discontinuity plays an important role in the quantitative analysis of this paper.

"Weak-inequality" and "strict-inequality" jobs. In this section, we analyze the impact on the economy when the magnitude of aggregate fluctuations, Δ_{Φ_p}, increases from 0 to an arbitrarily small number. When considering non-trivial values for Δ_{Φ_p}, it does not matter whether the inequality in the effort constraint is a weak or a strict inequality, unless there happens to be point mass exactly at $\phi_p = \chi$, which we rule out by assumption. When considering arbitrarily small values for Δ_{Φ_p}, however, it does matter whether the effort constraint is written with a weak or a strict-inequality.[12] We remove the ambiguity by assuming that there are two

[12] Consider a job with $\phi_p = \chi$ and an arbitrarily small drop in $\Phi_{p,t}$ starting at $\Phi_{p,t} = 1$. If the constraint is written as $\phi_p\Phi_{p,t} \geq \chi$, then the job satisfies the constraint before the drop in $\Phi_{p,t}$, but not after. If the constraint is written as $\phi_p\Phi_{p,t} > \chi$, then this same job satisfies the constraint

types of jobs. There is one type of job that has to satisfy the effort constraint with a *weak* inequality (as in equation 6) and one type of job that has to satisfy the effort constraint with a *strict* inequality.[13] This is made precise in the following definition.

Definition 2 *For each combination of ϕ_c and ϕ_p, there is one type of job with an effort constraint given by*

$$\phi_p \Phi_{p,t} \geq \chi \tag{10}$$

and one type of job with an effort constraint given by

$$\phi_p \Phi_{p,t} > \chi. \tag{11}$$

Jobs with $\phi_p = \chi$ facing the effort constraint given in equation (10) are referred to as "weak-inequality" jobs. Similarly, jobs facing the effort constraint given in equation (11) are referred to as "strict-inequality" jobs.

Whenever $\phi_p \neq \chi$, then there is no difference between the two types of jobs. Consequently, we only use the terms weak-inequality and strict-inequality when $\phi_p = \chi$. Moreover, even when $\phi_p = \chi$, then these two types of jobs only differ when $\Phi_{p,t} = 1$. In particular, both jobs satisfy the effort constraint whenever $\Phi_{p,t} > 1$ and both do not satisfy the effort constraint whenever $\Phi_{p,t} < 1$. Thus, both are fragile jobs. This setup with both weak and strict-inequality constraints not only gets rid of the ambiguity in writing the effort constraint. It also ensures that the case with arbitrarily small business cycles covers all the ways through which non-trivial business cycles affect fragile jobs, i.e., as indicated in figure 2. In particular, strict-inequality jobs are similar to the fragile jobs with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ in the gain area and weak-inequality jobs are similar to the fragile jobs with $\phi_p > \tilde{\phi}_{p,\text{no-bc}}$ in the loss or permanent loss area (depending on their value for ϕ_c).

Assumption 1

$$\mu = \hat{\mu}\phi_p$$

This assumption greatly simplifies the algebra.[14] What matters quantitatively for the results are the unemployment benefits of those jobs that cannot overcome the agency problem in a boom. These jobs have similar ϕ_p values. Consequently, the value of $\mu = \widehat{\mu}\phi_p$ does not vary much across those jobs for which the level of unemployment benefits matter.

neither before nor after the drop.

[13] The distribution of ϕ_c and ϕ_p is assumed to be the same for both types of jobs.

[14] If μ is proportional to ϕ_p, then the different cut-off levels for ϕ_c as a function of ϕ_p would no longer be parallel lines, as displayed in figures 1 and 2, but would all start at the origin and increase with a different slope.

Assumption 2

$$\begin{array}{ll} \textit{(i)} & 0<\beta<1,\ 0<\rho<1,\ 0<\pi<1,\ \phi_c\geq 0 \\ \textit{(ii)} & \frac{\left(1+\Delta_{\Phi_p}\right)\chi-\hat{\mu}\chi}{1-\beta\rho\pi}<\frac{\chi-\hat{\mu}\chi}{1-\beta\rho} \\ \textit{(iii)} & \frac{\chi-\mu}{\chi}=\frac{\chi-\hat{\mu}\chi}{\chi}>\Delta_{\Phi_p} \end{array} \quad . \tag{12}$$

The first part of the assumption simply ensures that parameters do not take on nonsensical values. The second part affects fragile jobs. It ensures that the NPV of the sequence of surplus values a job is expected to generate during one single stretch of high Φ_+ values does not exceed the NPV of the sequence of surplus values this job is expected to generate over its natural life if there are no business cycles to end it prematurely. The condition could be violated if booms make jobs extraordinarily productive. This is a weak assumption.[15] The third part of this assumption requires that the key variable of the agency problem, χ, is not too close to the value of μ. This assumption simplifies the analysis and ensures that the agency problem remains relevant for determining cut-off values over the business cycles for all affected jobs.

Cost of arbitrarily small business cycles for individual jobs. Cut-off levels are continuous functions of Δ_{Φ_p}. Consequently, arbitrarily small business cycles can only affect jobs *at* the cut-off values for ϕ_p or ϕ_c. The following proposition describes the result for timed-entry jobs.[16]

Proposition 1 ***Welfare impact for timed-entry jobs at the no-business-cycles cut-off levels when*** $\Delta_{\Phi_p}\rightarrow 0$***.*** *Suppose that (i)* $\phi_c=\tilde{\phi}_{c,no\text{-}bc}$*, (ii)* $\phi_p>\tilde{\phi}_{p,bc}(\Phi_-)$*, (iii)* $\omega_e=1$*, and (iv) part (i) of assumption 2 holds. Then*

$$\lim_{\Delta_{\Phi_p}\longrightarrow 0} L(\phi_c,\phi_p,\Delta_{\Phi_p})=0.$$

If the job creation decision is efficient, then all timed-entry jobs benefit from the presence of business cycles, that is, $L(\phi_c,\phi_p,\Delta_{\Phi_p})<0$. The reason is that for timed-entry jobs the job creation decisions in a world with business cycles *could* be the same as the decisions made in a world without business cycles, that is, always create the job when $\phi_c\leq\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ and never create the job when $\phi_c>\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$.

[15] To see that this is a weak assumption, suppose that $\beta=0.99$, $\rho=0.95$, and $\pi=0.875$. For these parameter values, the condition is satisfied as long as the surplus in a boom is not more than 298% above the surplus value in a world without business cycles.

[16] Proofs of the propositions are given in appendix B.

These benefits are proportional to Δ_{Φ_p}.[17] Consequently, they approach zero as the magnitude of business cycles approaches zero. Since agents are risk neutral and these jobs face no frictions, it is not surprising that $L(\cdot)$ is negative-valued and a smooth function.

The next proposition describes the impact of arbitrarily small business cycles for fragile jobs. Proposition 3 in appendix A, gives the more cumbersome formulas for the impact of business cycles of non-trivial magnitude.

Proposition 2 ***Welfare impact for fragile jobs at the no-business-cycles cut-off when*** $\Delta_{\Phi_p} \to 0$. *Suppose that assumptions 1 and 2 are satisfied. Then the change in welfare due to introducing arbitrarily small business cycles is given by the following expressions.*

(i) For weak-inequality jobs with $\phi_p = \tilde{\phi}_{p,no\text{-}bc}$, $\lim_{\Delta_{\Phi_p} \longrightarrow 0} L(\phi_c, \phi_p, \Delta_{\Phi_p})$ *is given by*

$$\begin{cases} \frac{1-\hat{\mu}}{2} + \frac{\phi_c}{\phi_p}\left(\frac{1-\beta\rho\pi-2(1-\beta\rho)}{2}\right) > 0 & \textit{if } 0 \leq \phi_c \leq \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) \\ 1-(1-\beta\rho)\frac{\phi_c}{\phi_p} - \hat{\mu} > 0 & \textit{if } \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) < \phi_c < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \\ 0 & \textit{if } \phi_c \geq \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \end{cases}$$

(ii) For strict-inequality jobs with $\phi_p = \tilde{\phi}_{p,no\text{-}bc}$, $\lim_{\Delta_{\Phi_p} \longrightarrow 0} L(\phi_c, \phi_p, \Delta_{\Phi_p})$ *is given by*

$$\begin{cases} \frac{\hat{\mu}-1}{2} + \frac{\phi_c}{\phi_p}\left(\frac{1-\beta\rho\pi}{2}\right) < 0 & \textit{if } 0 \leq \phi_c \leq \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) \\ 0 & \textit{if } \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) < \phi_c < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \\ 0 & \textit{if } \phi_c \geq \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \end{cases}$$

(iii) The total impact of arbitrarily small changes on a weak and a strict-inequality job is given by

$$\lim_{\Delta_{\Phi_p} \longrightarrow 0} L(\phi_c, \phi_p, \Delta_{\Phi_p}) = \begin{cases} \frac{\phi_c}{\phi_p}\beta\rho(1-\pi) = 0 & \textit{if } \phi_c = 0 \\ \frac{\phi_c}{\phi_p}\beta\rho(1-\pi) > 0 & \textit{if } 0 < \phi_c \leq \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) \\ 1-(1-\beta\rho)\frac{\phi_c}{\phi_p} - \hat{\mu} > 0 & \textit{if } \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) < \phi_c < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \\ 0 & \textit{if } \phi_c \geq \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \end{cases}$$

[17] In particular, when $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$, then

$$L(\phi_c, \phi_p, \Delta_{\Phi_p}) = -\frac{(1-\beta\rho)}{2(1-\beta\rho\pi)}\Delta_{\Phi_p}.$$

In contrast to the results for timed-entry jobs, the consequences of business cycles for affected jobs do not approach zero as the magnitude of business cycles become arbitrarily small. The reason is that business cycles—even if they are very small—affect whether jobs can or cannot satisfy the effort constraint. And this aspect generates a discrete change in the agents' welfare. The importance of this discontinuity will be discussed in detail in section 5.2.

For the discussion that follows, it may be helpful to remember that the idea behind the setup with weak and strict-inequality jobs is to have one job that just does and one job that just does not satisfy the effort constraint in a world without business cycles, but that have the same value for ϕ_p. In a world with business cycles both types of jobs satisfy the effort constraint in a boom, but not in a recession.

If $\phi_c = 0$, then weak (strict)-inequality jobs are negatively (positively) affected by business cycles. The combined impact is zero, since the extra surplus gained by a strict-inequality job in a boom is exactly offset by the surplus lost by a weak-inequality job in a recession, whereas the increased frequency of job creation does not carry a cost if $\phi_c = 0$. If $0 < \phi_c \leq \tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$, then the gains of the strict-inequality job are outweighed by the losses of the weak-inequality jobs resulting in a net loss.[18] For this case, ϕ_c remains low enough to warrant job entry whenever the effort constraint is satisfied. Since $\phi_c > 0$, it matters how often job creation costs are being paid. The strict-inequality job pays job creation costs more often in a world with than in a world without business cycles, since these jobs are never created in a world without business cycles. Whether weak-inequality jobs pay job creations costs more often depends on parameter values.[19] The last part of the proposition makes clear, however, that the *combined* effect on a weak and strict-inequality job is an unambiguous loss. In a world without business cycles, one of the two jobs pays the entry cost at the beginning of time, namely the weak-inequality job. In a world with business cycles, both jobs pay the entry cost at the beginning of time in a boom and neither do in a recession. The expected initial cost is, thus, the same. The expected job duration is shorter, however, in a world with business cycles. Consequently, job creation costs are paid more often.

If $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right) < \phi_c < \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$, then there is no longer a gain of the strict-inequality jobs to partially offset the losses of the weak-inequality jobs. If $\phi_c = \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$, then job creation could occur in a world without business cycles, but the benefits are the same as not creating the job, which is the outcome in the world with business cycles. Thus, there are no welfare consequences. There are also none when $\phi_c > \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$.

[18] Recall that the value of ϕ_p is the same for a weak and a strict-inequality job.

[19] The dependence is clear by considering extreme parameter values. In particular, if $\rho = 1$ ($\rho = 0$), then job creation costs are paid more (less) often in a world with business cycles.

3.3 The impact of typical business cycles

If aggregate fluctuations are *not* arbitrarily small, then the range of affected jobs increases. This is true for fragile and for timed-entry jobs. However, the case described above with arbitrarily small values for Δ_{Φ_p} already describes the different ways in which business cycles can affect jobs.

Fragile jobs. If Δ_{Φ_p} is arbitrarily small, then only jobs with $\phi_p = \chi$ are fragile jobs. By contrast, for non-trivial values of Δ_{Φ_p}, jobs with a range of values for ϕ_p are fragile, namely jobs with a value of ϕ_p such that $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$. Jobs with a value of ϕ_p such that $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\text{no-bc}}$ are jobs that never satisfy the effort constraint in a world without business cycles; these jobs are affected by business cycles in exactly the same way as strict-inequality jobs in the discussion above. That is, business cycles are beneficial for these jobs since the occasionally higher value of $\Phi_{p,t}$ allows them to satisfy the effort constraint when that happens. Jobs with a value of ϕ_p such that $\tilde{\phi}_{p,\text{no-bc}} \leq \phi_p < \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$ are jobs that always satisfy the effort constraint in a world without business cycles; these jobs are affected by business cycles in exactly the same way as weak-inequality jobs in the discussion above. That is, business cycles are harmful for these jobs because in the presence of business cycles these jobs either never operate or operate only during recessions.

There is one effect that is present for non-trivial values of Δ_{Φ_p} that is not present for arbitrarily small values. If Δ_{Φ_p} is arbitrarily small, then the output level of jobs that satisfy the effort constraint during a boom is equal to the output level of jobs that no longer satisfy the effort constraint during a recession. That output level is equal to χ. The output gained thus offsets the output lost. For non-trivial values of Δ_{Φ_p}, the output levels of jobs that start producing during a boom is *less* than χ while the output levels of jobs that stop producing during a recession is *more* than χ. But quantitatively this effect is small. The reason is that fluctuations in $\Phi_{p,t}$ are not very large, which means that $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{p,t})$ does not vary that much, which in turn implies that there is not that much variation in the value ϕ_p in between the two cut-off points. However, if there are cyclical jobs with values of ϕ_p above $\widetilde{\phi}_{p,\text{no-bc}}$, but no cyclical jobs with values of ϕ_p below $\widetilde{\phi}_{p,\text{no-bc}}$, then this would break the symmetry and increase the cost of business cycles. When calibrating the model, we assume, however, that the mass of jobs below $\widetilde{\phi}_{p,\text{no-bc}}$ is not smaller than the mass of jobs above, so we abstract from this possible reason for costly business cycles.

Timed-entry jobs. When $\Delta_{\Phi_p} > 0$, then there is a band of timed-entry jobs around the job creation cost cut-off level, $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, namely those jobs for which $\tilde{\phi}_c(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_c(\phi_p, \Phi_+)$. These jobs are created during booms, but not during recessions. Jobs with a value of ϕ_c such that $\tilde{\phi}_{c,\text{no-bc}}(\phi_p) < \phi_c < \tilde{\phi}_c(\phi_p, \Phi_+)$

benefit from business cycles because the higher value of $\Phi_{p,t}$ during a boom makes entry worthwhile. The higher the value of ϕ_c, the lower the value of this benefit. Jobs with a value of ϕ_c such that $\tilde{\phi}_c\left(\phi_p, \Phi_-\right) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$ also benefit from business cycles. For these jobs, the welfare loss of a recession is not as high as the welfare gain of a boom. The reason is that these jobs lower the consequences of a recession by delaying job creation. The lower the value of ϕ_c, the lower the benefit of paying ϕ_c in the future.

3.4 The impact of business cycles with inefficient job creation.

In the discussion above, we assumed that the entrepreneur's share of the surplus, ω_e, is equal to 1, which is a sufficient condition for job creation to be efficient. By focusing on efficient job creation, we made clear that our reason for costly business cycles does not depend on job creation being inefficient. As ω_e gets smaller, then the cut-off levels in figure 2 decrease proportionally. Qualitatively, the picture, thus, remains the same and we can still identify the same types of affected jobs. There are some differences regarding welfare consequences, however, especially for timed-entry jobs.

Inefficient job creation and timed-entry jobs. Above we argued that business cycles have a positive effect on timed-entry jobs. The reason is that the option to delay job creation during a recession creates value to the entrepreneur. If job creation is efficient, then this must also create value to the relationship as a whole. If job creation is not efficient, then the decision to delay still creates value for the entrepreneur. But this is not necessarily beneficial for the worker, since the entrepreneur ignores the benefits that accrue to the worker. Consequently, if ω_e is sufficiently low, then business cycles are costly for timed-entry jobs with a value of ϕ_c such that $\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$. These jobs are always immediately created in a world without business cycles, whereas creation is delayed during recessions in a world with business cycles. Business cycles remain beneficial for timed-entry jobs with a value of ϕ_c such that $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) < \phi_c \leq \tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)$. These jobs are never created in a world without business cycles, but they are created during a boom in a world with business cycles. If job creation is inefficient, then the relationship as a whole would benefit from job creation. Consequently, the creation during booms in a world with business cycles is welfare improving even though a further improvement would be possible if job creation would not be postponed during recessions.

Inefficient job creation and fragile jobs. The cut-off level for ϕ_c relative to ϕ_p is smaller for lower values of ω_e. This is important for the cost of business cycles for the following reasons. For cyclical fragile jobs, job creation costs are paid more often in a world with business cycles. Consequently, lower values of ϕ_c relative to ϕ_p lower the cost of business cycles. For permanent-loss fragile jobs, job creation costs are not paid at all anymore in a world with business cycles. This aspect dampens the negative impact of having permanent-loss fragile jobs. But this positive aspect is small when ϕ_c is low relative to ϕ_p. In our numerical examples, this second effect tends to dominate so that a decrease in ω_e increases the cost of business cycles.

4 Calculating the cost of business cycles

In this section, we describe the procedure used to calculate the cost of business cycles. The calibration procedure is based on US data and is described in section 4.1. We use US data to calibrate the model. The second subsection describes the steps taken to calculate the cost of business cycles. Appendix C contains the derivation of the formulas.

4.1 Calibration procedure

We start by presenting our choice for commonly used parameters. Next, we describe additional assumptions made and calibration targets. Some targets are not easily pinned down by empirical measures. An example, is the upper bound on the distribution of ϕ_c, $f\left(\phi_c|\phi_p\right)$. In our calibration, there are four such model characteristics for which the values of the empirical counterpart are uncertain. Although we have some information about their values, it is not precise enough to choose values with sufficient confidence. Consequently, we present results for a wide range of values for these uncertain targets.

4.1.1 Values for standard parameters

The period is a quarter and we set the value of β equal to 0.99. The value for μ indicates the value generated by an inactive job. Shimer (2005) uses a value for "not working" that is equal to 40% of market production. But his measure refers to all benefits that an unemployed worker receives, whereas here μ indicates the value produced by the inactive worker himself. As our benchmark, we assume that half of the number used by Shimer (2005) consists of actual net benefits generated by an unemployed worker. That is, we assume that $\mu = \hat{\mu}\phi_p$ with $\hat{\mu} = 0.2$. The value used by Shimer (2005) is considered to be too low by some.[20] Hall (2006) estimates the flow value of leisure forgone to be equal to 43%, and we consider this as an alternative estimate. We follow Krusell and Smith (1998) and set π equal to 0.875, which means that the expected duration of a boom and a recession is equal to eight

[20]See Mortensen and Nagypál (2007) for a discussion.

quarters, and we set Δ_{Φ_p} equal to 0.01.[21] According to the BLS, the median years of tenure for US workers 25 years and over varied between 4.7 and 5.2 years from 1996 to 2010. We set $\rho = 1 - \ln(2)/20 = 0.9653$, which would imply a median job duration of 20 quarters *if* no jobs are affected by business cycles. In our model, some jobs' duration is shortened by business cycles. The necessary adjustment depends on the particular case considered, but is never large.[22] Moreover, the adjustment would mean using a higher value for ρ and the higher the value for ρ the higher the cost of business cycles. Consequently, our choice for ρ is a conservative one.

4.1.2 Additional assumptions and calibration targets

A reduction in the entrepreneur's share, ω_e, reduces the cut-off levels for ϕ_c and, thus, average job creation costs. In the literature, there are some estimates for average job creation costs and we will use these estimates to determine ω_e. Unfortunately, the estimates vary quite a bit. Therefore, we consider this statistic as one of the four uncertain ones. The estimate based on Silva and Toledo (2009) implies a job creation cost of 1.245 times quarterly output.[23] By contrast, Shimer (2005) reports a value of only 0.157% of quarterly output.[24] This number is not based on direct evidence, but calibrated to fit model characteristics. We calculate the cost of business cycles for both estimates of the average job creation costs.

Determining the joint density $f(\phi_c, \phi_p)$ would be difficult. As mentioned above, we do not even have accurate information about the mean of ϕ_c. Moreover, what matters for our quantitative results is not some rough global description of the

[21] We checked the assumptions regarding the duration using HP-filtered residuals for GDP. Within our sample from 1947Q1 to 2010Q4, we find that from 1949Q1 to 2008Q3 there are sixteen complete recessions (periods with negative HP-filtered residuals surrounded by positive residuals) and sixteen complete booms (periods with positive HP-filtered residuals surrounded by negative values). The average durations are equal to 7.1 and 7.9 quarters for recessions and booms, respectively. This corresponds roughly to the assumption adopted here that the expected duration of a boom is roughly equal to the expected duration of a recession. Moreover, this corresponds closely to an expected duration of 8 quarters adopted by Krusell and Smith (1998).

[22] To see that the downward bias is small, suppose that *all* of the observed fluctuations in the extensive employment margin are due to (low-duration) fragile jobs and none to (high-duration) timed-entry jobs. Under this assumption, we get the largest possible bias. This assumption implies that 0.0776 (which is—as discussed below—our estimate for the change in employment over the business cycle) is equal to $\frac{E_{\text{C-fragile}}}{E_{\text{C-fragile}}+E}$. These jobs exist half of the time. The median job duration is then equal to the value of the cumulative distribution function at $0.50-0.0776/2$, which is equal to 17.82 quarters.

[23] This consists of two parts. First, the total cost of hiring a person is 3.6% of the quarterly wage, which corresponds to 2.4% of quarterly output using a standard labor share value. Second, productivity losses and training costs during the first year are 31% of output in each quarter. The discounted value of this cost is equal to $\sum_{i=0}^{3} 0.31\beta^i = 1.2215$.

[24] In particular, the flow cost of vacancy posting is 0.213 and the vacancy filling probability is 1.355.

distribution, but information about the mass in relatively small areas, including information about jobs that only operate in the world we do not live in, that is, the world without business cycles. Also, strictly speaking we need the joint distribution of ϕ_c, ϕ_p, *and* χ, because one would think that χ is not the same for all jobs. Instead of specifying a distribution for the complete distribution of job specific characteristics, we take a stand on just a limited set of aspects of the distribution and combine this with an estimate for *observed* employment fluctuations along the extensive margin. This turns out to be enough to calculate the cost of business cycles.

The observed standard deviation of detrended employment is equal to 3.88%.[25] We use this statistic to pin down the mass of cyclical fragile jobs that are destroyed in a recession. In our model, these detrended employment fluctuations correspond to cyclical fragile jobs as well as timed-entry jobs that are exogenously destroyed during the recession and not being recreated. If there would be no timed-entry jobs, then the mass of fragile jobs would be $2 \times 3.88\% = 7.76\%$, since a 7.76% difference in the recession and boom employment levels gives a standard deviation of 3.88%. To take the behavior of timed-entry jobs into account, we do the following. The expected duration of a downturn is equal to 8 quarters. Consequently, to get the mass of fragile jobs we subtract from 7.76% the expected number of timed-entry jobs that disappears in eight quarters.

We make the following two assumptions regarding the distribution of ϕ_c and ϕ_p.

Assumption 3

$$\int_{\tilde{\phi}_{p,bc}(\Phi_+)}^{\tilde{\phi}_{p,no\text{-}bc}} f\left(\phi_c, \phi_p\right) d\phi_p \leq \int_{\tilde{\phi}_{p,no\text{-}bc}}^{\tilde{\phi}_{p,bc}(\Phi_-)} f\left(\phi_c, \phi_p\right) d\phi_p.$$

Sufficient for this inequality to hold is that the density $f\left(\phi_p|\phi_c\right)$ is non-decreasing for values of ϕ_p such that $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \leq \phi_p \leq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$. Since $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+)$ and $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$ are likely to be located in the left tail of the distribution, the density is unlikely to be decreasing around these values.

Assumption 4 *$f\left(\phi_c|\phi_p\right)$ is uniformly distributed on $\left[\underline{\phi}_c\left(\phi_p\right), \overline{\phi}_c\left(\phi_p\right)\right]$.*

Both the lower bound and the upper bound of $f\left(\phi_c|\phi_p\right)$ are important parameters in determining the cost of business cycles. For example, if the upper bound is sufficiently low, then there are no permanent-loss fragile jobs. These bounds are the

[25] We use US total nonfarm employment from 1948Q1 to 2007Q4. If the sample is extended to 2010Q4, then this number increases to 0.0487. We use a linear trend to detrend the data, that is, we calculate the cost of business cycles relative to the case in which there is a constant growth rate.

second and third of our four uncertain targets. For both bounds we consider two quite different values.

Regarding the upper bound, $\overline{\phi}_c\left(\phi_p\right)$, our preferred choice is to assume that the upper bound is *not less* than $\tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$, i.e., the cut-off level if job duration is not shortened by business cycles. If the upper bound of the distribution would be less than this cut-off value, then there are no jobs at all that are too costly to be created. This seems impossible. Consequently, our preferred value for $\overline{\phi}_c\left(\phi_p\right)$ is a value not less than $\tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$. Since jobs above the highest cut-off level, $\tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$, are not affected by business cycles, it does not matter whether we set $\overline{\phi}_c\left(\phi_p\right)$ equal to $\tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$ or to any higher level. For simplicity, we set $\overline{\phi}_c\left(\phi_p\right) = \tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$.

The assumption that $\overline{\phi}_c\left(\phi_p\right) \geq \tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$ implies that all possible permanent-loss fragile jobs are in the domain of the distribution. As an alternative, we set $\overline{\phi}_c\left(\phi_p\right)$ such that half of all permanent-loss fragile jobs do not exist, that is, we set $\overline{\phi}_c\left(\phi_p\right)$ equal to the average of the cut-off level when job duration is and when it is not affected by the agency problem, $\frac{1}{2}\left(\tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right) + \tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)\right)$.

Regarding the lower bound we also consider two quite different values. The first is 0. This means that there are jobs that can be created at no or little cost. This may not be realistic. As an alternative, we consider $\underline{\phi}_c\left(\phi_p\right) = \frac{1}{2}\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$, which halves the mass of cyclical fragile jobs.

Besides assumption 3, which is a very weak assumption, we only make one additional assumption about the distribution of ϕ_p. That is, we assume that the productivity of cyclical fragile jobs relative to the productivity of jobs not affected by the agency problem, i.e., jobs such that $\phi_p \geq \tilde{\phi}_{p,\mathrm{bc}}\left(\Phi_-\right)$, takes on a certain value. This is the fourth of our four uncertain targets. We consider two values. The first is based on the assumption that non-fragile workers are on average as productive as college graduates and fragile workers are on average as productive as those without a college degree. This would imply that fragile workers are on average half as productive as non-fragile workers.[26] The property that fragile workers are only found in the bottom of the distribution of ϕ_p is not necessarily true if χ is not the same across all jobs. To simplify the exposition, we assumed that χ did not vary with ϕ_p but one can easily imagine that it does. Moreover, if fragile jobs are substantially less productive than other jobs, then the Solow residual would no longer be procyclical. This composition effect could be substantial.[27] Therefore, we also consider the case in which fragile jobs have on average the same productivity as non-fragile jobs and

[26] See Goldin and Katz (2008).

[27] For example, suppose that half of the observed employment fluctuations are due to fragile jobs and suppose that these are half as productive. Then the recession value of the Solow residual is $1 - \Delta_{\Phi_p} = 0.99$ and the boom value is only slightly higher, namely $\left(1 + \Delta_{\phi_p} + (0.0776/2)\left(1 + \Delta_{\Phi_p}\right)/2\right) / (1 + 0.0776/2) = 0.9911$, where 0.0776 is our estimate for the total change in employment over the business cycle.

fragility is due to a high value of χ. In this case, the Solow residual is equal to $\Phi_{p,t}$.

No need to calibrate χ. Our calibration procedure does not require taking a stand on the value for χ. Moreover, our calibration procedure allows the underlying model to be such that χ is not the same for all jobs. We consider this a big advantage, since there is no clear empirical counterpart for χ. Moreover, although we have presented our model using one particular agency problem, our reasoning carries over to different types of agency problems, for which χ would have a different interpretation.[28] We will discuss this in more detail below and in appendix C, but the reason why we do not have to take a stand on the distribution of χ is roughly the following. Instead of taking a stand on the distribution of χ, ϕ_c, and ϕ_p across jobs and to directly calculate the mass of cyclical and permanent-loss fragile jobs, we use the assumptions made above on the distribution of ϕ_c and ϕ_p to directly calculate the mass of timed-entry jobs relative to the mass of regular jobs, i.e., non-fragile jobs that are also not timed-entry jobs. Timed-entry jobs are not affected by χ and we do not need information about χ to calculate this fraction. Given the amount of observed employment adjustment along the extensive margin and given the calculated mass of timed-entry jobs, we can then determine the mass of cyclical fragile jobs as the residual between the two. Given the mass of cyclical fragile jobs and given our assumption on $f\left(\phi_c|\phi_p\right)$, it is then straightforward to calculate the mass of permanent-loss fragile jobs.

4.2 Steps to calculate cost of business cycles

Calculating the cost of business cycles is quite tedious. Here we give an intuitive description and refer the reader to appendix C for precise formulas. The procedure calculates the cost of business cycles for the three affected groups, cyclical fragile jobs, permanent-loss fragile jobs, and timed-entry jobs. The calculation for each group consists of two steps. First, we calculate the output level associated with this group. That is we calculate $\int\int \phi_p f\left(\phi_c \phi_p\right) d\phi_c d\phi_p$ where the integration is over the $\left(\phi_c, \phi_p\right)$ pairs that characterize the group. Given this output measure, it is relatively straightforward to calculate the cost of business cycles. The reason is that the cost of business cycles is a function of just this output measure and known structural parameters, such as β, ρ, and π. To understand the discussion below, it is important to realize that the cut-off values for ϕ_c only depend on known structural parameter values, but do not depend on the distribution $f\left(\phi_c,\phi_p\right)$.

Timed-entry jobs. We first calculate the mass of timed-entry jobs and the associated output level. To identify the mass of timed-entry jobs, we do the following.

[28] In appendix A.2 of Den Haan and Sedlacek (2009), we work out the case in which the agency problem is related to firm financing.

Timed-entry jobs have the same output level as those jobs for which the job creation costs is lower (so that they are not timed-entry jobs). Given our assumption on $f\left(\phi_c|\phi_p\right)$ and knowledge of the cut-off levels, we can calculate the mass of timed-entry jobs relative to regular jobs, i.e., jobs that are nor timed-entry nor fragile jobs. Given that they have the same output level as jobs with lower job creation costs, we can also calculate the output level associated with timed-entry jobs relative to the output level of jobs with lower job creation costs.

Cyclical fragile jobs. In the first step, we calculate the output that is earned by cyclical fragile jobs during a boom. That is, the output associated with the jobs in the "loss" and the "gain" area in figure 2. There are two elements in this step. First, from the observed employment variation along the extensive margin we subtract that part of the calculated mass of timed-entry jobs that is destroyed during a typical downturn. This gives the mass of cyclical fragile jobs. Second, using the assumption on the relative productivity of fragile jobs, we calculate the output that can be produced by cyclical fragile jobs relative to a measure of aggregate output. This way we obtain a measure for the mass of cyclical fragile jobs and their joint output level without having to specify a distribution for χ. That is, instead of obtaining a direct measure for the mass of cyclical fragile jobs, we get it indirectly from a measure for observed detrended employment variation and the calculated mass of timed-entry jobs.

Permanent-loss fragile jobs. Calculating the output that permanent-loss fragile jobs would produce in a world without business cycles is tricky, since these jobs are not observed in the world we live in, that is, the one with business cycles. We calculate the mass of these jobs as follows. Above, we showed how to calculate the mass of *all* cyclical fragile jobs. Using assumption 3, we can get a lower bound on the mass of cyclical fragile jobs with a value of ϕ_p that exceeds $\tilde{\phi}_{p,\text{no-bc}}$. Permanent-loss fragile jobs also have a value of ϕ_p that exceeds $\tilde{\phi}_{p,\text{no-bc}}$. Using the assumption on $f\left(\phi_c|\phi_p\right)$ and knowledge of the cut-off levels for ϕ_c, we can thus calculate the mass of permanent-loss fragile jobs relative to regular jobs. As documented in figure 2, we know that permanent-loss fragile jobs have the same output level as cyclical fragile jobs with a value of ϕ_p that exceeds $\tilde{\phi}_{p,\text{no-bc}}$. This allows us to calculate the output associated with the permanent-loss fragile jobs relative to aggregate output.

5 Quantitative impact of business cycles

In section 5.1, we report and discuss estimates for the cost of business cycles according to our model. In section 5.2, we discuss the role of the agency problem and compare the effort constraint (and the discontinuity it induces) with other types of constraints that may limit market production.

5.1 Results

Table 2 reports the welfare cost of business cycles. In Lucas (1987), the welfare cost of business cycles is estimated to be less than 0.1% of aggregate consumption when agents are risk averse and the coefficient of relative risk aversion is equal to 10.[29] By contrast, the numbers in table 2 vary from 2.03% to 12.7% of aggregate output.

Each block of cells in the table has three rows. The number in the top row reports the total cost of business cycles as a fraction of total output. The three numbers in the second row indicate the cost due to permanent-loss fragile jobs, the cost due to cyclical fragile jobs, and the cost due to timed-entry jobs. The number in the bottom row indicates the entrepreneur's share of the surplus, ω_e, which is set so that the average job creation cost is equal to the target level.

Consider the four blocks in the top-left corner. These four blocks correspond to the case when the relative productivity of fragile jobs takes on the low value and average job creation costs take on the high value. The four blocks differ in what is assumed about the lower bound and the upper bound of the distribution of job creation costs ϕ_c and, thus, differ regarding the importance of permanent-loss fragile and cyclical fragile jobs.

Suppose that the lower bound of the distribution for the job creation cost level, ϕ_c, is equal to zero and the upper bound is equal to the highest cut-off level for ϕ_c, i.e., $\tilde{\phi}_c\left(\phi_p, \Phi_+\right)$. The cost of business cycles is then equal to 3.18%. Only a small part, namely 0.05%, is due to cyclical fragile jobs. A gain of 0.61% due to extra jobs being created during expansions is roughly offset by a loss of 0.66% due to some jobs no longer being able to operate without interruption. Job creation costs are paid more often for these jobs so the gain and the loss do not offset. But job creation costs are quantitatively not that important even at the high end of the estimate for job creation costs. For timed-entry jobs, there is a gain of 0.0018%. This consists of a gain of 0.1189% due to jobs with a value of ϕ_c above $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$ producing in a world with and not producing in a wold without business cycles and a loss of 0.1171% due to jobs with a value of ϕ_c below $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$ delaying entry during downturns. The latter benefits entrepreneurs, but hurts workers.

The 3.18% cost of business cycles is, thus, almost completely due to permanent-loss fragile jobs. That is, business cycles are costly, because jobs that are valuable from a social welfare point of view are no longer created in the presence of business

[29] The costs of business cycles according to the Lucas formula are equal to $0.5\sigma_c^2\gamma$, where σ_c is the standard deviation of the cyclical component of aggregate consumption and γ is the coefficient of relative risk aversion. Thus, if σ_c is equal to 0.013 and γ is equal to 10, then the costs of business cycles are equal to 0.084%.

Table 2: Business cycle costs

	high average job creation costs						low average job creation costs					
	$\overline{\phi}_c = \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$			$\overline{\phi}_c < \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$			$\overline{\phi}_c = \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$			$\overline{\phi}_c < \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$		
relative productivity of cyclical jobs equal to 0.5												
		3.18%			2.03%			3.39%			2.10%	
$\underline{\phi}_c = 0$	3.13%	0.05%	−0.00%	1.92%	0.11%	–	3.40%	−0.01%	−0.00%	2.10%	−0.00%	–
		0.14			0.22			0.02			0.03	
		6.17%			4.05%			6.55%			4.21%	
$\underline{\phi}_c = 1/2\widetilde{\phi}^*_{c,bc}$	6.10%	0.07%	−0.00%	3.91%	0.14%	–	6.55%	−0.00%	−0.00%	4.21%	−0.00%	–
		0.12			0.18			0.02			0.02	
relative productivity of cyclical jobs equal to 1												
		6.17%			3.89%			6.57%			4.03%	
$\underline{\phi}_c = 0$	6.07%	0.10%	−0.00%	3.68%	0.21%	–	6.59%	−0.02%	−0.00%	4.04%	−0.01%	–
		0.14			0.22			0.02			0.03	
		11.97%			7.78%			12.70%			8.10%	
$\underline{\phi}_c = 1/2\widetilde{\phi}^*_{c,bc}$	11.84%	0.13%	−0.00%	7.51%	0.27%	–	12.71%	−0.01%	−0.00%	8.10%	−0.00%	–
		0.12			0.18			0.02			0.02	

Notes: The number in the first row is the *total* cost of business cycles as a fraction of GDP. The numbers in the second row are the cost due to permanent-loss fragile jobs, cyclical fragile jobs, and timed-entry jobs. A hyphen indicates that there are no timed-entry jobs. A negative number means that business cycles are beneficial. The number in the third row is the value of ω_e needed to get average job creation costs equal to its target value. Also, "high" ("low") average job creation costs corresponds to 1.245 (0.157) times quarterly output. When $\overline{\phi}_c < \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$, the value of $\overline{\phi}_c$ is set equal to $\frac{1}{2}\left(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) + \tilde{\phi}^*_{c,\mathrm{bc}}(\phi_p, \Phi_+)\right)$. This means that the area of permanent-loss fragile jobs is cut in half.

cycles.

The numbers are based on the assumption that the lower bound of the distribution for ϕ_c, $\underline{\phi}_c$, is equal to 0, which means that there are jobs that are very cheap to create. We also considered a higher value for this lower bound. Increasing the lower bound increases the cost of business cycles from 3.18% to 6.17%. Everything else equal, the increase in the lower bound of ϕ_c cuts the area with cyclical fragile jobs in half. However, cyclical fragile jobs do not contribute that much to the cost of business cycles. So how can the cost of business cycles be so sensitive to changes in $\underline{\phi}_c$? Key in understanding this result is that we do not keep the distribution $f\left(\phi_c, \phi_p\right)$ fixed in our calibration procedure. Instead, we use *observed* employment fluctuations to pin down the mass of cyclical fragile jobs. To understand what is going on, suppose that there are no timed-entry jobs. The change in $\underline{\phi}_c$ would then have no effect on the mass of cyclical fragile jobs whatsoever, since the mass of cyclical fragile jobs would remain equal to the observed adjustment in employment along the extensive margin. The reason for the increase in the cost of business cycles is the following. If the same number of cyclical fragile jobs is located in a smaller area of $\left(\phi_c, \phi_p\right)$ pairs, then this implies that there are more permanent-loss fragile jobs, since the area of permanent-loss fragile jobs is not affected by the change in $\underline{\phi}_c$. The sensitivity of the results to the value of $\underline{\phi}_c$ means that $\underline{\phi}_c$ is a key parameter. Without having a reliable estimate for this lower bound, it makes sense to be conservative and to focus on the lowest possible value, i.e., $\underline{\phi}_c = 0$. But if the lower bound is higher, then the cost of business cycles could very well be substantially higher.

So far, we have assumed that the upper bound of the distribution of ϕ_c, $\overline{\phi}_c$, is equal to $\tilde{\phi}_c\left(\phi_p, \Phi_+\right)$. Recall that the results would be the same if $\overline{\phi}_c$ would exceed $\tilde{\phi}_c\left(\phi_p, \Phi_+\right)$. We have more confidence in setting $\overline{\phi}_c \geq \tilde{\phi}_c\left(\phi_p, \Phi_+\right)$, then in setting $\underline{\phi}_c = 0$. The reason is that a lower value for the upper bound implies that there would be no jobs at all that are too costly to be created, except possibly for fragile jobs. This sounds too good to be true. But it is instructive to see what happens if we consider a lower value for $\overline{\phi}_c$. A reduction of $\overline{\phi}_c$ from $\tilde{\phi}_c\left(\phi_p, \Phi_+\right)$ to the alternative lower value cuts the *area* of permanent-loss jobs in half. But this does not cut the *number* of permanent-loss jobs in half for the following reason. The reduction in $\overline{\phi}_c$ eliminates all timed-entry jobs. If there are no timed-entry jobs left, then all observed employment fluctuations must be due to cyclical fragile jobs. A higher mass of cyclical fragile jobs implies a higher mass of cyclical permanent-loss fragile jobs.[30] The cost due to permanent-loss fragile jobs reduces from 3.13% to 1.92%, that is, by less than half. Reassigning observed employment fluctuations from timed-

[30]In our calibration procedure, if there are more fragile jobs with a value of ϕ_c below $\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$, then there must be more fragile jobs with a value of ϕ_c above $\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p, \Phi_+\right)$, conditional on the value of ϕ_p.

entry jobs to cyclical fragile jobs increases the cost due to cyclical fragile jobs from 0.05% to 0.11%. The disappearance of timed-entry jobs also eliminates the positive effect of business cycles due to timed-entry jobs. But this is a very small effect.

Sensitivity to average job creation costs. The case considered up to now is based on the high estimate for job creation costs, namely 1.245% of quarterly output. The implied value for the entrepreneur's share of the surplus, ω_e, ranged from 0.14 to 0.22. This is just the reward for the entrepreneurial activity of creating a job and financing job creation. It is not a reward for financing capital goods. Thus, the implied values for ω_e seem high. If we consider the low value for the average job creation costs, i.e., 0.157% of quarterly output, then the value for ω_e drops considerably to values that are at most 0.0274. Even though the change in the average job creation costs and the change in the value of ω_e are substantial, the implied cost of business cycles varies relatively little. For example, when $\underline{\phi}_c = 0$ and $\overline{\phi}_c = \tilde{\phi}_c\left(\phi_p, \Phi_+\right)$, then the cost of business cycles increases from 3.18% to 3.39%. In figure 2, a reduction of ω_e would induce a proportional shift of all the cut-off levels for ϕ_c. Moreover, given our calibration procedure, the mass in the relevant areas remains the same. The reason that the cost of business cycles increases is that the cost of job creation relative to the level of output has gone down. This matters for the following reasons. An offsetting benefit of business cycles for permanent-loss fragile jobs is that these jobs no longer have to pay job creation costs. This benefit is smaller when job creation costs are smaller. By contrast, for cyclical fragile jobs the reduction in the cost of job creation costs *reduces* the cost of business cycles, because these jobs pay job creation costs more often in the presence of business cycles. The first effect clearly dominates.[31]

Efficient job creation. Recall that job creation is efficient when $\omega_e = 1$, i.e., when the entrepreneur has all the bargaining power. At this value for ω_e, average job creation costs are at least 5.7 times quarterly output, which is implausible. Nevertheless, it is interesting to see whether business cycles are costly if job creation is efficient. They clearly are. For example, when $\underline{\phi}_c = 0$ and $\overline{\phi}_c = \tilde{\phi}_c\left(\phi_p, \Phi_+\right)$, then the total cost of business cycles is equal to 1.71% of GDP. The division across the three groups, relative to the results reported above for lower ω_e values, is a bit different. The cost endured by cyclical fragile jobs is now equal to 0.46%. Thus, the relative importance of cyclical jobs for the total cost of business cycles has increased. The reason is that business cycles induce these jobs to pay job creation costs more

[31] At the low value for the average job creation costs considered, the gains of cyclical fragile jobs that start producing during booms is virtually equal to the losses of the cyclical fragile jobs that stop producing during recessions. That is, job creation costs have become so low that the actual payment do not play much of a role anymore. They still matter because the affect the entrepreneur's decision to create jobs.

often and this negative effect is costlier when job creation costs are higher. When the job creation decision is efficient, then timed-entry jobs are marginal jobs from a social welfare point of view. Consequently, a bit more or a bit less entry has even less of an impact than when job creation is not efficient.[32] In particular, the gain of business cycles due to timed-entry jobs is now 0.0010%. With inefficient entry the numbers are roughly 100 times larger, but go in opposite directions, i.e. a gain for timed-entry jobs above the no-business-cycles cut-off level and a loss for timed-entry jobs below this cut-off. The net result is then similar to the net result with efficient entry.

More productive fragile jobs. The numbers above are based on the assumption that fragile jobs are only half as productive as other jobs. Fragile jobs are an important part of employment fluctuations over the business cycle. Consequently, the Solow residual could be acyclical or even countercyclical, because the jobs created during booms are less productive than the other jobs. As an alternative, we consider the (extreme) case when fragile jobs are as productive as other jobs. This means that fragility is no longer due to a job having a low value for ϕ_p but due to having a high value for χ.

The results are reported in the lower half of table 2. Not surprisingly, the cost of business cycles are now much higher, roughly twice as high. This is solely due to increased cost of business cycles for permanent-loss fragile and for cyclical fragile jobs, since the cost for timed-entry jobs is not affected by this change.

Higher unemployment benefits. Finally, we consider an increase in the value of unemployment from 20% to 43% of market output. Recall that the value of unemployment only includes the benefits from a social welfare point of view and excludes transfers. Consequently, 43% is a substantial number. Not surprisingly, this increase lowers the cost of business cycles. But the costs of business cycles remain substantial. For the same cases considered in table 2, the cost of business cycles varies between 1.41% and 7.96%.

Cost of business cycles for individuals. Since we assume that agents are risk neutral, business cycles only affect the wellbeing of an individual if this individual's decisions are affected. This is a relatively small group. Regarding jobs that produce in the world we live in, that is, the one with business cycles, this is limited to jobs that are part of the observed employment adjustment over the business cycle along the extensive margin. Regarding jobs that do not produce in the world we live in,

[32] Recall that with efficient entry the gain of business cycles due to timed-entry jobs goes to zero as the magnitude of business cycles approaches zero.

i.e., permanent-loss fragile jobs, this is still a relatively small group because the importance of this group is linked to the importance of cyclical fragile jobs.

Given that only a relatively small group of individuals is affected by business cycles, the cost of business cycles can only be substantial if the consequences for affected individuals are sizeable.

This is made precise in table 3, which reports the individual cost of business cycles for the different affected groups. Each block of cells has three rows. The number in the first row is the individual cost of business cycles averaged across the permanent-loss fragile jobs. The two numbers in the second row are the average individual costs for cyclical fragile jobs with a value of ϕ_p below and above χ, that is, for the jobs that gain and loose compared to the world with no business cycles. The two numbers in the third row are the average individual costs for timed-entry jobs with a value of ϕ_c below and above $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$.

To understand the numbers consider a *hypothetical* job for which job creation costs are zero and that always produces in a world without business cycles, but never in a world with business cycles. Since the value of not producing market production is equal to 20% of market production, the cost of business cycles as a fraction of market production would be equal to 80% for this hypothetical job. Being completely driven out of business by business cycles only happens for permanent-loss fragile jobs. These jobs have non-zero job creation costs and an offsetting benefit of business cycles is that these job creation costs do not have to be paid. Consequently, 80% is an upper bound for the cost of business cycles. Similarly, 80% is an upper bound for the gain of a timed-entry job that is created during booms in a world with business cycles and is never created in a world without business cycles. The productivity of fragile jobs relative to other jobs does not matter for the individual cost of business cycles, since this cost is calculated relative to the individual's own productivity.

Consider the block of cells in the top-left corner. This corresponds to the case with the high average job creation costs. Moreover, $\underline{\phi}_c = 0$ and $\overline{\phi}_c = \tilde{\phi}_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$. The table shows that the results are similar for alternative values. The cost of business cycles for permanent-loss fragile jobs is on average equal to 72.9% of market production. Not having to pay job creation costs pulls the cost a bit below the worst case scenario of a loss of 80%, but not by much. Cyclical fragile jobs that start producing during booms in a world with business cycles gain on average 37.7% and cyclical fragile jobs that can no longer produce during recessions in a world with business cycles incur on average a loss of 40.8%. Timed-entry jobs for which entry is delayed during recessions in a world with business cycles face a cost of business cycles equal to 9.25%. Part of the net cost is a very small gain for the entrepreneur for whom the option to postpone entry is beneficial. Timed-entry jobs that are created during booms in a world with business cycles but are never created in a world without business cycles face a gain of business cycles equal to 59.7%.

Table 3: Individual costs

	high average job creation costs				low average job creation costs			
	$\overline{\phi}_c = \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$		$\overline{\phi}_c < \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$		$\overline{\phi}_c = \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$		$\overline{\phi}_c < \widetilde{\phi}_{c,bc}(\phi_p, \Phi_+)$	
	72.91%		72.04%		79.11%		79.00%	
$\underline{\phi}_c = 0$	−37.69%	40.78%	−36.09%	41.51%	−40.15%	39.66%	−39.94%	39.75%
	9.25%	−59.65%	–	–	10.56%	−68.04%	–	–
	73.77%		73.45%		79.21%		79.17%	
$\underline{\phi}_c = 1/2\widetilde{\phi}^*_{c,bc}$	−36.80%	41.19%	−35.06%	41.98%	−40.03%	39.71%	−39.81%	39.81%
	9.43%	−60.81%	–	–	10.58%	−68.19%	–	–

Notes: This table reports the cost of business cycles for individual jobs as a fraction of individual output, ϕ_p. A negative number means that business cycles are beneficial. The number in the first row gives the cost of business cycles for permanent-loss fragile jobs averaged across all jobs in this category. The numbers in the second row give the average cost of business cycles for cyclical fragile jobs that gain, i.e., jobs for which $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) < \phi_p < \tilde{\phi}_{p,\mathrm{no\text{-}bc}}$, and the average cost for jobs that loose, i.e, jobs for which $\tilde{\phi}_{p,\mathrm{no\text{-}bc}} \leq \phi_p < \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$. The number in the third row gives the average cost of business cycles for timed entry jobs when $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) < \phi_c \leq \phi_{c,\mathrm{no\text{-}bc}}(\phi_p)$ and the average cost when $\phi_{c,\mathrm{no\text{-}bc}}(\phi_p) < \phi_c \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$. Also, "high" ("low") average job creation costs corresponds to 1.245 (0.157) times quarterly output. When $\overline{\phi}_c < \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$, the value of $\overline{\phi}_c$ is set equal to $\frac{1}{2}\left(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) + \tilde{\phi}^*_{c,\mathrm{bc}}(\phi_p, \Phi_+)\right)$. This means that the area of permanent-loss fragile jobs is cut in half.

The gain for jobs with a value of ϕ_c above $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ exceeds the loss for jobs with a value of ϕ_c above $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ since workers gain much more from becoming employed during expansions (and remaining employed until exogenous severance) then they loose from delayed hiring during recessions.[33]

5.2 The role of the agency problem

The agency problem creates a discontinuity in the revenues produced by agents. That is, a small change in the value of $\phi_p \Phi_{p,t}$ can create a much bigger drop in revenues, namely from a value equal to or above χ to a value equal to μ. This discontinuity is essential in generating non-trivial cost of business cycles. The reason is the following.

Fluctuations in $\Phi_{p,t}$ are small, which makes it difficult for business cycles to matter quantitatively. In our model, fluctuations in $\Phi_{p,t}$ only affect agents if changes in $\Phi_{p,t}$ affect employment adjustment along the extensive margin. In our calibration procedure, the extent to which fluctuations in $\Phi_{p,t}$ affect the extensive margin is constrained by observed changes in employment along the extensive margin. This is a non-trivial number, but still only a very small part of the population. Even though business cycles do not affect many agents in our model, they are costly because the agents that are affected are affected a lot.[34]

In our experience, the following alternative models have turned out to be helpful in making clear the tight link in our model between facing an agency problem and the discontinuity and thus the role of the agency problem in generating large cost of business cycles. Before starting the discussion, the reader is reminded that the agency problem by itself does not cause business cycles to be costly. Over the business cycle, the agency problem has a discontinuous negative effect if $\Phi_{p,t}$ falls but also has a discontinuous positive effects if $\Phi_{p,t}$ increases. For business cycles to be costly, we also need job creation costs to be positive.

5.2.1 Model with only a participation constraint

Suppose that there is no effort constraint.[35] There still would be a participation constraint, since the worker would need to get at least μ. Thus, the effort constraint of equation (6) is replaced by

$$\phi_p \Phi_{p,t} \geq \mu. \tag{13}$$

[33] There are, however, more timed-entry jobs below than above $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, which means that the consequences of business cycles for the two types of timed-entry jobs aggregated are closer to each other in absolute magnitude.

[34] Another (but related) way to think about this is the following. The cut-off levels are continuous functions of $\Phi_{p,t}$. Consequently, small changes in $\Phi_{p,t}$ will not affect a lot of jobs unless the distribution is such that there is a high concentration of jobs exactly around the cut-off levels.

[35] In Den Haan and Sedlacek (2009), we describe in detail the model without the effort constraint.

For simplicity, we focus on the case with efficient job creation. Business cycles turn out to be beneficial for the following reason. In a world with business cycles, the job creation and the continuation decisions *could* be identical to those in a world without business cycles. The reason is that equation (13) is not a feasibility constraint, like the effort constraint, but an optimality condition. If the decisions could be the same, then the revenues would be more volatile in a world with business cycles, but—given our assumption of risk neutrality—this would not affect utility levels. Consequently, business cycles cannot make agents worse off. In fact, business cycles would be welfare enhancing.

The reader may find the discussion surprising, because the participation constraint given by equation (13) seems identical to the effort constraint of equation (6). The difference lies in what is being earned if the relevant constraint is not satisfied. Consider a job with a value of ϕ_p that is exactly at the cut-off level in a world without business cycles, i.e., $\phi_p = \tilde{\phi}_{p,\text{no-bc}} = \mu$. The equations are simpler for this particular job, but the argument carries over to all fragile jobs. Expected revenues in a world without business cycles, $R_{\text{no-bc}}$, and expected revenues in a world with business cycles, R_{bc}, are given by

$$\begin{aligned} R_{\text{no-bc}} &= \mu, \\ R_{\text{bc}} &= \text{E}\left[\max\left\{\mu, \mu\Phi_{p,t}\right\}\right] = \mu\text{E}\left[\max\left\{1, \Phi_{p,t}\right\}\right]. \end{aligned} \tag{14}$$

Clearly, $R_{\text{bc}} > R_{\text{no-bc}}$ when $\Delta_{\Phi_p} > 0$, since $\max\{1, \Phi_{p,t}\}$ is a convex function. Moreover, increased volatility in $\Phi_{p,t}$ would increase the value of R_{bc}. That is, without the agency problem the unemployment benefit works like a put option and increased uncertainty raises the expected payoff. If effort constraint (6) holds instead, then expected revenues are given by

$$\begin{aligned} R_{\text{no-bc}} &= \chi, \\ R_{\text{bc}} &= \tfrac{1}{2}\mu + \tfrac{1}{2}\chi\Phi_{+}. \end{aligned} \tag{15}$$

$R_{\text{no-bc}} > R_{\text{bc}}$, unless the values of μ and χ are close to each other.[36] Business cycles in the presence of the agency problem, thus, creates an unavoidable drop in revenues because $\mu < \chi$. This makes the problem very different from the one with the participation constraint as in equation (13).

5.2.2 Adjustment costs

Suppose that output is given by $\phi_p\Phi_{p,t}n_t$, where n_t is a flexible labor input. In addition, suppose that output net of adjustment costs is given by $\phi_p\Phi_{p,t}n_t^{\alpha} - \zeta\left(n_t - n_{t-1}\right)^2$. One might think that business cycles are costly if ζ is high, so that a model with a high value for ζ would mimic the discontinuous payoff structure in our model.

[36] The precise parameter restriction we use is given in assumption 2.

But note that firms can avoid the high adjustment costs by simply not adjusting n_t. This would imply that the actual employment is not equal to the optimal level of employment. But for realistic movements in $\Phi_{p,t}$, such deviations cannot be very large.

Now suppose that there is a cost when the level of output is adjusted. In particular, suppose that output net of adjustment costs is given by $\phi_p \Phi_{p,t} n_t^\alpha - \zeta \left(\phi_p \Phi_{p,t} n_t^\alpha - \phi_p n_{\text{no-bc}}^\alpha\right)^2$. If the firm cannot adjust n_t, then business cycles could be costly, but only if ζ is very high, since fluctuations in $\Phi_{p,t}$ are small. In this case, the firm is forced to vary the output level even though it is costly. The question arises why small fluctuations in output would correspond with high adjustment cost. Moreover, the cost could be avoided if the firm can adjust the value of n_t. In this case, the firm can avoid the adjustment costs in output by adjusting n_t such that $\phi_p \Phi_{p,t} n_t$ remains constant. Since fluctuations in $\Phi_{p,t}$ are small, this would not require large changes in n_t.

5.2.3 Minimum-output requirement

Consider a model in which firms have to keep output above a certain minimum level, ζ, for example, because of regulation or because it is too expensive to reduce capacity. Again, suppose that firm-level output is given by $\phi_p \Phi_{p,t} n_t$. Firms can ensure that output exceeds ζ by adjusting n_t. Also suppose that $\mu < \zeta$. This setup in which firm output has to exceed ζ seems similar to the setup with the agency problem in which firms can only operate if their output exceeds χ. In fact, it is quite different. Let $n^* (\Phi_-)$ be the optimal level of n if there is no minimum-output requirement. Consider jobs such that $\phi_p \Phi_- n^* (\Phi_-) < \zeta$. The difference with the agency problem is that these jobs are not necessarily discontinued and, thus, do not necessarily face a discontinuous drop in revenues down to μ. The reason is that the option exists to increase n above $n^* (\Phi_-)$ so that the minimum-output requirement is still satisfied. If $\phi_p \Phi_{p,t}$ does not change very much, then the adjustment in n_t needed to satisfy the minimum-output requirement cannot be very costly. Consequently, business cycles cannot be very costly.

5.2.4 Avoiding the agency problem

The idea behind an agency problem is that adjustment to avoid the agency problem is *not* possible. Of course, faced with a discontinuous drop in income after a job destruction, agents would have an incentive to try to avoid the agency problem.[37]

[37] This is a standard aspect of models with agency problems. For example, in models in which an agency problem affects firm financing, there is an incentive of the firm owner to accumulate so much wealth that the agency problem affecting outside financing is no longer relevant. To insure that the agency problem remains relevant, there must be something that goes against this incentive. This could be a tax advantage of debt or impatient entrepreneurs.

In our environment in which only the entrepreneur faces an effort constraint, the entrepreneur could avoid the agency problem by accumulating enough funds. She can then pay the worker in advance so that the decision to put in low effort would no longer affect participation by the worker. But this would not alleviate the agency problem if the worker also has the option to put in low effort, as is the case in the original contractual fragility framework of Ramey and Watson (1997).[38] Creative readers may think of other ways to avoid the agency problem. But the premise of this paper is that agency problems *are* important for business cycle analysis for one reason or another. This paper shows that if this is the case *and* job creation costs are positive, then business cycles are costly.

6 Related literature

Following the classic Lucas (1987) paper, there have been numerous attempts to develop models in which business cycles are costly.[39] One strand of the literature considers preferences in which fluctuations are more harmful to the agent.[40] A second strand of the literature considers the possibility that risk is not spread evenly across agents. When idiosyncratic risk is persistent, then this line of research generates estimates for the cost of business cycles that are an order of magnitude larger than those found by Lucas.[41] The idea is that unemployment has very negative consequences for the individual and is a relatively rare event. Business cycles can be costly if risk sharing among agents is sufficiently limited and the agents' degree of risk aversion is high enough.

Lucas' calculations on the cost of business cycles are based on a comparison between two economies: one with and one without business cycles, but both have the same long-run growth path. But the presence of business cycles could very well have long-term level or even growth effects. Empirical evidence for this view can be found in Ramey and Ramey (1995), Martin and Rogers (2000), Loayza, Ranciere, Serven, and Ventura (2007), Burnside and Tabova (2009), and Den Haan and Sedlacek (2009).[42] Although the relationship seems stronger for low and middle income countries, the link also exists for OECD countries.

Our paper is related to a set of papers that build theoretical models in which

[38] It also requires that the entrepreneur and the owner of the firm are the same person.

[39] See Lucas (2003) for a summary.

[40] Examples of this line of research are Alvarez and Jermann (2005) and Tallarini (2000).

[41] See Storesletten, Telmer, and Yaron (2001), Krebs (2007), Santis (2007), and Krusell, Mukoyama, Sahin, and Smith (2009).

[42] These papers establish an unconditional negative correlation between business cycles and real activity. There are also channels that generate a positive correlation. For example, Levchenko, Ranciere, and Thoenig (2009) show that financial liberalization leads to an increase in both volatility and expected economic growth.

fluctuations have such level or growth effects. As shown in Ramey and Ramey (1991), Jones, Manuelli, and Stacchetti (2000), Epaulard and Pommeret (2003), and Barlevy (2004), there is a relationship between growth and volatility in endogenous growth models. Barlevy (2004) points out that for a reduction in volatility to have a quantitatively important effect on output, it is important that the increase in investment induced by a reduction in volatility not only increases the growth rate of consumption, but also does *not* lead to an initial reduction in the level of consumption.[43] Barlevy (2004) accomplishes this by introducing diminishing returns to investment into an endogenous growth model. The nonlinearity makes it possible for a reduction in fluctuations to have a positive effect on the growth rate, even if average investment levels, and thus the initial consumption level, are not affected by fluctuations.

Such a nonlinearity is also important in papers that show that volatility has a negative effect on the average *level* of real activity. Gali, Gertler, and Lopez-Salido (2007) consider a simple New-Keynesian model in which the efficiency losses due to mispricing in a recession are not offset by the efficiency gains in a boom. Business cycles are then welfare reducing, although the effects turn out to be small. Jung and Kuester (2011) and Hairault, Langot, and Osotimehin (2010) show that the matching model contains a nonlinearity that causes volatility in job finding rates to reduce average unemployment. In particular, increases in job finding rates during booms have less of an impact than decreases in job finding rates during recessions, because the unemployment rate is smaller during booms than recessions.

How does our explanation compare with the ones given in these papers? Our main mechanism is *not* that expansions and contractions have asymmetric/nonlinear effects on the job creation decision or on the agency problem. That is, if we would add a third middle state to our model in which Φ_p is equal to $\Phi_0 \equiv \frac{1}{2}(\Phi_+ + \Phi_-)$, then there is no robust reason for asymmetric effects if Φ_p increases from Φ_0 to Φ_+ or decreases from Φ_0 to Φ_-.[44] That is, a boom is still pretty much the opposite of a recession. In our model the interaction between job creation costs and the agency problem permanently eliminate some jobs in the presence of business cycles. Business cycles have a negative level effect, but are themselves not asymmetric.

[43] Mertens (2008) shows how fluctuations can affect the average level of investment in a model without endogenous growth. In his model, agents have distorted beliefs and fluctuations worsen this distortion. Consequently, the risk premium is higher and the average capital stock is lower in a world with business cycles.

[44] The model could generate asymmetries if the mass of projects is distributed in an asymmetric manner.

7 Robustness and concluding comments

In this paper, we have presented a model in which business cycles are costly, because they affect the average level of output. In developing the model, we had to make several choices about, for example, the particular agency problem agents face. In this last section, we contend that the mechanism highlighted in this paper is robust to changing many of the particular choices made.

We adopted the contractual fragility framework of Ramey and Watson (1997). The relevant aspects are the following. First, the relationship has to generate enough resources to overcome the agency problem. Second, if the relationship does not overcome the agency problem, then it breaks up and the resources generated are substantially less than if it could have continued. These two features are common to many other agency problems. In appendix A.2 of Den Haan and Sedlacek (2009), we work out the case in which firm financing is hampered by an agency problem. The implications are very similar.

In our model, business cycles make some jobs temporarily or permanently impossible, which is costly since these jobs have a positive surplus from a social welfare point of view. But the exclusive focus on the extensive employment margin may underestimate the cost of business cycles. Business cycles have no welfare consequences for jobs that are not temporarily or permanently eliminated. But the mechanism highlighted in this paper may be relevant for these jobs as well. This would be the case if existing jobs have the possibility to make investments to increase the job's productivity. The cost of these investments would be like the job creation cost. Combined with an agency problem (say a hold up problem) that is associated with the extra revenues, then we have the same features used in our job creation framework.

We assumed that idiosyncratic differences are permanent. This simplifies the analysis enormously. But we do not think that it matters for the mechanism. What matters is that there always are jobs that belong to the critical regions. If there are jobs in these regions, then business cycles lower the average level of output. It does not matter whether the jobs and workers in these regions are always the same ones or whether they can be different ones as time passes.

In our model, the distribution for ϕ_c and ϕ_p is taken as exogenous, but the results could be affected if agents can affect their own productivity and/or level of job creation costs. This may be the case if workers and entrepreneurs affected by business cycles could put in search or another type of effort to get different values for ϕ_c and/or ϕ_p. But business cycles would remain costly as long as it is not that easy to make marginal workers more productive or reduce job creation costs.

We end the paper with the following observation. Our numerical work focuses on the cost of business cycles. But our time-varying shock, $\Phi_{p,t}$, does not have to be an aggregate shock. It also could be a sectoral, geographical, or an idiosyncratic shock. These fluctuations are likely to be larger than business cycle fluctuations. If the cost of business cycles is already non-trivial, then the welfare consequences of these other fluctuations could very well be a staggering number.

A Formulas and additional results

In this appendix, we do the following. First, we give the formulas that determine the cut-off levels. Second, we give the expressions for the welfare cost of business cycles for individual jobs for generic values for Δ_{Φ_p}. Third, we show that among affected jobs the jobs at the cut-off levels are affected the least (the most) for those jobs that are negatively (positively) affected by business cycles. Proofs are given in appendix B.

A.1 Timed-entry jobs.

Cut-off levels for timed-entry jobs, $\tilde{\phi}_{c,\mathbf{bc}}\left(\phi_p, \Phi_{p,t}\right)$. Recall that $N_{\mathrm{bc}}(\phi_c, \phi_p, 1, \Phi_{p,t})$ and $N_{\mathrm{bc}}(\phi_c, \phi_p, 0, \Phi_{p,t})$ stand for the NPVs of a job with job creation cost ϕ_c, productivity ϕ_p, aggregate productivity $\Phi_{p,t}$ when the job creation cost has and has not been paid, respectively. The value of $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-)$ satisfies the condition that during a recession the entrepreneur is indifferent between waiting until $\Phi_p = \Phi_+$ and immediate entry. That is, it can be solved from the following system:

$$N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-) = \mu + \beta \begin{bmatrix} \pi N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-)+ \\ (1-\pi) N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_+) \end{bmatrix}, \tag{16a}$$

$$N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-) = -\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) + \phi_p \Phi_- \tag{16b}$$
$$+\beta \begin{bmatrix} \pi\rho \left(N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) \right) + \\ \pi(1-\rho) N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-)+ \\ (1-\pi)\rho \left(N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_+) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) \right) + \\ (1-\pi)(1-\rho) N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_+) \end{bmatrix},$$

$$N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_+) = -\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) + \phi_p \Phi_+ \tag{16c}$$
$$+\beta \begin{bmatrix} \pi\rho \left(N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_+) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) \right) + \\ \pi(1-\rho) N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_+)+ \\ (1-\pi)\rho \left(N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) \right) + \\ (1-\pi)(1-\rho) N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_-) \end{bmatrix}.$$

These equations use that $N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 1, \Phi_{p,t}) = N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), \phi_p, 0, \Phi_{p,t}) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-)$, that is, waiting does not create any additional value at $\phi_c = \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-)$

in both aggregate states. The first equation specifies the benefits of waiting with job creation. The second equation specifies the benefits of immediate creation. The last equation defines the additional term showing up in the first two equations. Recall that next period, the aggregate state stays the same with probability π and the job faces an exogenous destruction shock with probability $1-\rho$.

The cut-off value in a boom, $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$, can be solved from the condition that the entrepreneur is indifferent between creating and not creating the project.

$$\frac{\mu}{1-\beta} = -\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) + \phi_p \Phi_+ \tag{17a}$$

$$+\beta \left[\begin{array}{c} \pi\rho\left(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) + \frac{\mu}{1-\beta}\right) + \\ \pi(1-\rho) \times \frac{\mu}{1-\beta} + \\ (1-\pi)\rho N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+), \phi_p, 1, \Phi_-) + \\ (1-\pi)(1-\rho) \times \frac{\mu}{1-\beta} \end{array} \right]$$

$$N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+), \phi_p, 1, \Phi_-) = \phi_p \Phi_- \tag{17b}$$

$$+\beta \left[\begin{array}{c} \pi\rho N_{\mathrm{bc}}(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+), \phi_p, 1, \Phi_-) + \\ \pi(1-\rho) \times \frac{\mu}{1-\beta} + \\ (1-\pi)\rho\left(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) + \frac{\mu}{1-\beta}\right) + \\ (1-\pi)(1-\rho) \times \frac{\mu}{1-\beta} \end{array} \right]$$

The first equation equates the benefits of not creating the job to the benefits of creating the job. The second equation defines the additional term that shows up in the first equation.

NPVs of timed-entry jobs. The NPVs $N_{\mathrm{bc}}(\phi_c, \phi_p, 0, \Phi_+)$, $N_{\mathrm{bc}}(\phi_c, \phi_p, 1, \Phi_+)$, $N_{\mathrm{bc}}(\phi_c, \phi_p, 0, \Phi_-)$, and $N_{\mathrm{bc}}(\phi_c, \phi_p, 1, \Phi_-)$ are solved from the following system of equations.

$$
\begin{aligned}
N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_+) &= -\phi_c + \phi_p \Phi_+ \\
&\quad + \beta \begin{pmatrix} \pi \rho N_{\text{bc}}(\phi_c, \phi_p, 1, \Phi_+) \\ +\pi (1-\rho) N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_+) \\ +(1-\pi) \rho N_{\text{bc}}(\phi_c, \phi_p, 1, \Phi_-) \\ +(1-\pi)(1-\rho) N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_-) \end{pmatrix}, \qquad (18) \\
N_{\text{bc}}(\phi_c, \phi_p, 1, \Phi_+) &= N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_+) + \phi_c, \qquad (19) \\
N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_-) &= \mu + \beta \begin{pmatrix} \pi N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_-) \\ +(1-\pi) N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_+) \end{pmatrix}, \qquad (20) \\
N_{\text{bc}}(\phi_c, \phi_p, 1, \Phi_-) &= \phi_p \Phi_- \\
&\quad + \beta \begin{pmatrix} \pi \rho N_{\text{bc}}(\phi_c, \phi_p, 1, \Phi_-) \\ +\pi (1-\rho) N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_-) \\ +(1-\pi) \rho N_{\text{bc}}(\phi_c, \phi_p, 1, \Phi_+) \\ +(1-\pi)(1-\rho) N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_+) \end{pmatrix}. \qquad (21)
\end{aligned}
$$

Most affected timed-entry jobs. For timed-entry jobs, the following holds:

$$
L(\tilde{\phi}_{c,\text{no-bc}}, \phi_p, \Delta_{\Phi_p}) < L(\phi_c, \phi_p, \Delta_{\Phi_p}) \leq 0 \text{ for } \phi_c \neq \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right). \qquad (22)
$$

If job creation is efficient, then timed-entry jobs benefit from business cycles. Thus, the inequality means that jobs at the cut-off *benefit* the most. The intuition is straightforward. Jobs with a value of ϕ_c such that $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) < \phi_c < \tilde{\phi}_c\left(\phi_p, \Phi_+\right)$ benefit from business cycles because the higher value of $\Phi_{p,t}$ during a boom makes entry worthwhile. The higher the value of ϕ_c, the lower the value of this benefit. Jobs with a value of ϕ_c such that $\tilde{\phi}_c\left(\phi_p, \Phi_-\right) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$ benefit from business cycles since they create value by postponing job creation during a recession. The lower the value of ϕ_c, the lower the benefit of paying ϕ_c in the future.

Timed-entry jobs when $\omega_e < 1$. When $\omega_e < 1$, then the formulas for $N_{\text{bc}}(\cdot)$ stay the same, but it is no longer the case that $N_{e,\text{bc}}(\cdot) = N_{\text{bc}}(\cdot)$. Consequently, the cut-off levels adjust. To get the formulas for this case simply (i) replace in equations (16) and (17), μ (i.e., what the relationship can produce when not producing market production) with 0 (which is what the entrepreneur produces outside the relationship) and (ii) replace $\phi_p \Phi_{p,t}$ (i.e. what the relationship can produce) with $\omega_e\left(\phi_p \Phi_{p,t} - \mu\right)$ (i.e., what the entrepreneur gets).

A.2 Fragile jobs

The following proposition gives cut-off levels and cost of business cycles for fragile jobs.

Proposition 3 *Suppose that (i) assumption 2 is satisfied and (ii) $\omega_e = 1$. Then, the following properties hold.*

1. ***Cut-off levels for fragile jobs.***[45] *The cut-off level for ϕ_c if there are no business cycles is equal to*

$$\tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) = \frac{\phi_p - \mu}{1-\beta\rho} \text{ if } \phi_p \geq \tilde{\phi}_{p,no\text{-}bc}. \tag{23}$$

The cut-off level for ϕ_c if there are business cycles is as follows:

$$\tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) = \frac{\phi_p\left(1+\Delta_{\Phi_p}\right)-\mu}{1-\beta\rho\pi} \quad \text{if } \tilde{\phi}_{p,bc}\left(\Phi_+\right) \leq \phi_p < \tilde{\phi}_{p,bc}\left(\Phi_-\right). \tag{24}$$

Jobs that do not satisfy the effort constraint during recessions but do so in a world without business cycles have a lower cut-off level for ϕ_c in a world with than in a world without business cycles. That is,

$$\tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) \text{ if } \tilde{\phi}_{p,no\text{-}bc} \leq \phi_p < \tilde{\phi}_{p,bc}\left(\Phi_+\right). \tag{25}$$

Moreover, this gap does not approach zero as business cycles become arbitrarily small.

$$\lim_{\Delta_{\Phi_p}\longrightarrow 0} \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) = \frac{\phi_p - \mu}{1-\beta\rho\pi} < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right) = \frac{\phi_p - \mu}{1-\beta\rho} \tag{26}$$

2. ***Welfare impact for fragile jobs,*** $\Delta_{\Phi_p} >> 0$***.*** *If $\Delta_{\Phi_p} > 0$, then the change in welfare for weak-inequality jobs with $\phi_p = \tilde{\phi}_{p,no\text{-}bc}$ and fragile jobs with $\phi_p > \tilde{\phi}_{p,no\text{-}bc}$ is given by*

$$\begin{aligned} & L(\phi_c, \phi_p, \Delta_{\Phi_p}) \\ = & \begin{cases} \frac{1-\Delta_{\Phi_p}-\hat{\mu}}{2} + \frac{\phi_c}{\phi_p}\left(\frac{1-\beta\rho\pi-2(1-\beta\rho)}{2}\right) > 0 & \text{if } 0 \leq \phi_c < \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right), \\ 1-(1-\beta\rho)\frac{\phi_c}{\phi_p} - \hat{\mu} > 0 & \text{if } \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) \leq \phi_c < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right), \\ 0 & \text{if } \phi_c \geq \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right), \end{cases} \end{aligned} \tag{27}$$

where $\hat{\mu} = \mu/\phi_p$. The change in welfare for strict-inequality jobs with $\phi_p = \tilde{\phi}_{p,no\text{-}bc}$ and fragile jobs with $\phi_p < \tilde{\phi}_{p,no\text{-}bc}$ is given by

$$\begin{aligned} & L(\phi_c, \phi_p, \Delta_{\Phi_p}) \\ = & \begin{cases} \frac{\hat{\mu}-\left(1+\Delta_{\Phi_p}\right)}{2} + \frac{\phi_c}{\phi_p}\left(\frac{1-\beta\rho\pi}{2}\right) < 0 & \text{if } 0 \leq \phi_c < \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right), \\ 0 & \text{if } \tilde{\phi}^*_{c,bc}\left(\phi_p, \Phi_+\right) \leq \phi_c < \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right), \\ 0 & \text{if } \phi_c \geq \tilde{\phi}_{c,no\text{-}bc}\left(\phi_p\right). \end{cases} \end{aligned} \tag{28}$$

[45] Jobs with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ are not created, because they never satisfy the effort contraint in a world without business cycles.

Business cycles are beneficial for strict-inequality jobs and costly for weak-inequality jobs. To study the net effect on a strict-inequality and a weak-inequality job with the same values for ϕ_p and ϕ_c, we calculate the total effect. The total welfare loss for this pair at $\phi_p = \tilde{\phi}_{p,no\text{-}bc}$ is given by[46]

$$
\begin{aligned}
&L(\phi_c, \phi_p, \Delta_{\Phi_p}) \qquad (29)\\
= &\begin{cases}
\left(-\Delta_{\Phi_p} + \frac{\phi_c}{\phi_p}\beta\rho(1-\pi)\right) < 0 & \textit{if } 0 \le \phi_c < \frac{\phi_p \Delta_{\Phi_p}}{\beta\rho(1-\pi)} \\
\left(-\Delta_{\Phi_p} + \frac{\phi_c}{\phi_p}\beta\rho(1-\pi)\right) = 0 & \textit{if } \phi_c = \frac{\phi_p \Delta_{\Phi_p}}{\beta\rho(1-\pi)} \\
\left(-\Delta_{\Phi_p} + \frac{\phi_c}{\phi_p}\beta\rho(1-\pi)\right) > 0 & \textit{if } \frac{\phi_p \Delta_{\Phi_p}}{\beta\rho(1-\pi)} < \phi_c < \tilde{\phi}^*_{c,bc}(\phi_p, \Phi_+) \\
1 - (1-\beta\rho)\frac{\phi_c}{\phi_p} - \hat{\mu} > 0 & \textit{if } \tilde{\phi}^*_{c,bc}(\phi_p, \Phi_+) \le \phi_c < \tilde{\phi}_{c,no\text{-}bc}(\phi_p) \\
0 & \textit{if } \phi_c \ge \tilde{\phi}_{c,no\text{-}bc}(\phi_p)
\end{cases}
\end{aligned}
$$

Least affected fragile jobs. Among fragile jobs, the cost of business cycles are smallest at the cut-off value, i.e., when $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$. That is, for fragile jobs the following holds:

$$L(\phi_c, \tilde{\phi}_{p,\text{no-bc}}, \Delta_{\Phi_p}) < L(\phi_c, \phi_p, \Delta_{\Phi_p}) \text{ for } \phi_p \ne \tilde{\phi}_{p,\text{no-bc}}. \qquad (30)$$

For fragile jobs, the cost (gain) of business cycles are, thus, smallest (largest) for the jobs considered in the previous section. First, consider jobs with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$ and $\phi_c \le \tilde{\phi}^*_{c,\text{bc}}(\phi_p, \Phi_+)$. The entry cost of these jobs is low enough so that entry is profitable even though they only survive until the next recession. In a world with business cycles, these jobs generate output equal to $(1 + \Delta_{\Phi_p})\,\phi_p > \mu$ during a boom and μ during a recession, whereas they always generate μ in a world without business cycles. The larger the value of ϕ_p, the larger the gains (for equal values of ϕ_c). Thus, the welfare *gain* attained by strict-inequality jobs with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ is an *upper* bound for the gains achieved by fragile jobs with $\phi_p < \tilde{\phi}_{p,\text{no-bc}}$.

Next, consider fragile jobs with $\phi_p \ge \tilde{\phi}_{p,\text{no-bc}}$. In this case, the welfare *loss* attained by weak-inequality jobs with $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ are a *lower* bound for the losses suffered. Business cycles permanently reduce market output of fragile jobs when ϕ_c is above $\tilde{\phi}^*_{c,\text{bc}}(\phi_p, \Phi_+)$ and temporarily (namely during recessions) when ϕ_c is below $\tilde{\phi}^*_{c,\text{bc}}(\phi_p, \Phi_+)$. Consequently, the loss is larger when the value of ϕ_p is larger (for equal values of ϕ_c).

[46] If $\phi_c \le \phi^*_{c,\text{bc}}(\phi_p, \Phi_+)$, this is the sum of the weak-inequality and the strict-inequality job. If $\phi_c \le \phi^*_{c,\text{bc}}(\phi_p, \Phi_+) < \phi_c \le \tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, this is just equal to the effect of the weak-inequality job, since the strict-inequality job is not affected by business cycles.

Fragile jobs when $\omega_e < 1$. If $\omega_e < 1$, then the formulas have to be adjusted in the same way as they were adjusted above for timed-entry jobs.

B Proofs

Proof of proposition 1. This proposition focuses on jobs that can always overcome the efficiency condition and have a value of ϕ_c that is exactly at the boundary. Thus,

$$\phi_c = \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) \text{ and } \phi_p > \tilde{\phi}_{p,\text{bc}}\left(\Phi_-\right). \tag{31}$$

In a world without business cycles, the value of activating a job with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ would be equal to the value of not activating. Thus,

$$N_{\text{no-bc}}\left(\phi_c, \phi_p, 0\right) = \frac{\mu}{1-\beta} \text{ if } \phi_c = \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) \text{ and } \phi_p > \tilde{\phi}_{p,\text{bc}}\left(\Phi_-\right). \tag{32}$$

In a world with business cycles, jobs with $\phi_c = \tilde{\phi}_{c,\text{no-bc}}$ and $\phi_p > \tilde{\phi}_{p,\text{bc}}\left(\Phi_-\right)$ are timed-entry jobs. Timed-entry jobs are only created during expansions, but an already activated job continues operating in a recession. The NPVs for timed-entry jobs are given by the following equations:

$$\begin{aligned}
N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) &= -\phi_c + \phi_p\left(1+\Delta_{\Phi_p}\right) \qquad (33)\\
&\quad +\beta\begin{pmatrix} \pi\rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1+\Delta_{\Phi_p}) \\ +\pi\left(1-\rho\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) \\ +\left(1-\pi\right)\rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1-\Delta_{\Phi_p}) \\ +\left(1-\pi\right)\left(1-\rho\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) \end{pmatrix}, \\
N_{\text{bc}}(\phi_c, \phi_p, 1, 1+\Delta_{\Phi_p}) &= N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) + \phi_c, \qquad (34)\\
N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) &= \mu + \beta\begin{pmatrix} \pi N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) \\ +\left(1-\pi\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) \end{pmatrix}, \qquad (35)\\
N_{\text{bc}}(\phi_c, \phi_p, 1, 1-\Delta_{\Phi_p}) &= \phi_p\left(1-\Delta_{\Phi_p}\right) \qquad (36)\\
&\quad +\beta\begin{pmatrix} \pi\rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1-\Delta_{\Phi_p}) \\ +\pi\left(1-\rho\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) \\ +\left(1-\pi\right)\rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1+\Delta_{\Phi_p}) \\ +\left(1-\pi\right)\left(1-\rho\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) \end{pmatrix}.
\end{aligned}$$

Let

$$D\left(\phi_c, \phi_p\right) = N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) - N_{\text{bc}}(\phi_c, \phi_p, 1, 1-\Delta_{\Phi_p}) + \phi_c. \tag{37}$$

Then

$$\begin{aligned}
\mathrm{E}\left[N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_{p,t})\right] &= \frac{N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) + N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p})}{2} \qquad (38)\\
&= \frac{-\left(1-\beta\rho\right)\phi_c + \phi_p\left(1+\Delta_{\Phi_p}\right) + \mu - \beta\rho\left(1-\pi\right) D\left(\phi_c, \phi_p\right)}{2\left(1-\beta\right)}.
\end{aligned}$$

For these jobs the cut-off level for ϕ_c is given by

$$\tilde{\phi}_{c,\text{no-bc}} = \frac{\phi_p - \mu}{1 - \beta\rho}. \tag{39}$$

Using this last expression we get that

$$\mathrm{E}\left[N_{\text{bc}}(\tilde{\phi}_{c,\text{no-bc}}, \phi_p, 0, \Phi_{p,t})\right] = \frac{\phi_p \Delta_{\Phi_p} + 2\mu - \beta\rho\left(1-\pi\right) D\left(\phi_c, \phi_p\right)}{2\left(1-\beta\right)}. \tag{40}$$

The expression for D is calculated as follows. Working out the terms in the definition of D we get the following

$$\begin{aligned}
D\left(\phi_c, \phi_p\right) &= \phi_c + N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) - N_{\text{bc}}(\phi_c, \phi_p, 1, 1-\Delta_{\Phi_p}) && (41)\\
&= \phi_c + \mu + \beta\begin{pmatrix} \pi N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) \\ +\left(1-\pi\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) \end{pmatrix} \\
&\quad - \left(\phi_p\left(1-\Delta_{\Phi_p}\right) + \beta\begin{pmatrix} \pi\rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1-\Delta_{\Phi_p}) \\ +\pi\left(1-\rho\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) \\ +\left(1-\pi\right)\rho N_{\text{bc}}(\phi_c, \phi_p, 1, 1+\Delta_{\Phi_p}) \\ +\left(1-\pi\right)\left(1-\rho\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) \end{pmatrix}\right) \\
&= \phi_c + \mu + \beta\begin{pmatrix} \pi N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) \\ +\left(1-\pi\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) \end{pmatrix} \\
&\quad - \left(\phi_p\left(1-\Delta_{\Phi_p}\right) + \beta\begin{pmatrix} \pi\left(N_{\text{bc}}(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}) + \rho\left(\phi_c - D\right)\right) \\ +\left(1-\pi\right) N_{\text{bc}}(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}) + \rho\phi_c \end{pmatrix}\right) \\
&= \phi_c\left(1-\beta\rho\right) + \mu - \phi_p\left(1-\Delta_{\Phi_p}\right) + \beta\pi\rho D. && (42)
\end{aligned}$$

From this, we get

$$D\left(\phi_c, \phi_p\right) = \frac{\phi_c\left(1-\beta\rho\right) + \mu - \phi_p\left(1-\Delta_{\Phi_p}\right)}{1-\beta\pi\rho}. \tag{43}$$

Moreover,

$$D\left(\phi_c, \phi_p\right) = \frac{\phi_p \Delta_{\Phi_p}}{1-\beta\pi\rho} \text{ if } \phi_c = \tilde{\phi}_{c,\text{no-bc}} = \frac{\phi_p - \mu}{1-\beta\rho}. \tag{44}$$

If we combine this expression for $D\left(\phi_c, \phi_p\right)$ with the expression in equation (40) and the definition of the welfare loss, then we get that

$$\begin{aligned}
L\left(\phi_c, \phi_p, \Delta_{\Phi_p}\right) &= \frac{\left(1-\beta\right)}{\phi_p}\left(\frac{\mu}{1-\beta} - \frac{\phi_p \Delta_{\Phi_p} + 2\mu - \beta\rho\left(1-\pi\right) D\left(\phi_c, \phi_p\right)}{2\left(1-\beta\right)}\right) && (45)\\
&= -\frac{\Delta_{\Phi_p}\left(1-\beta\rho\right)}{2\left(1-\beta\pi\rho\right)}.
\end{aligned}$$

Proof of proposition 2. The expressions for the case with $\Delta_{\Phi_p} > 0$ are given in proposition 3. The results follow directly by letting Δ_{Φ_p} approach 0.

Part 1 of proposition 3. $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$ is defined as the value of ϕ_c at which the value of creating the job is equal to the value of never activating the job.[47] Thus,

$$N_{\text{no-bc}}(\tilde{\phi}_{c,\text{no-bc}}, \phi_p, 1) - \tilde{\phi}_{c,\text{no-bc}} = \frac{\mu}{1-\beta}. \tag{46}$$

The value of a created job is given by

$$N_{\text{no-bc}}(\phi_c, \phi_p, 1) = \phi_p + \beta\rho N_{\text{no-bc}}(\phi_c, \phi_p, 1) + \beta\left(1-\rho\right) N_{\text{no-bc}}(\phi_c, \phi_p, 0). \tag{47}$$

If assumption 2 is satisfied, then activating a job is profitable if the effort constraint is satisfied and the job creation costs are low enough. Thus,

$$N_{\text{no-bc}}(\phi_c, \phi_p, 1) = N_{\text{no-bc}}(\phi_c, \phi_p, 0) + \phi_c \text{ when } \left\{ \begin{array}{c} \phi_c \leq \tilde{\phi}_{c,\text{no-bc}} \text{ and} \\ \phi_p \geq \tilde{\phi}_{p,\text{no-bc}} \end{array} \right. . \tag{48}$$

Combining the last three equations gives

$$\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) = \frac{\phi_p - \mu}{1-\beta\rho} \text{ if } \phi_p \geq \tilde{\phi}_{p,\text{no-bc}}. \tag{49}$$

Next, we calculate the value of $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p\right)$, i.e., the cut-off level for fragile jobs. If the job creation costs are low enough, then these jobs are created in a boom, but not in a recession.[48] Thus, if $\phi_c \leq \tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$, then

$$\begin{aligned} & N_{\text{bc}}\left(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}\right) \\ & = \\ -\phi_c + \phi_p\left(1+\Delta_{\Phi_p}\right) + \beta & \left(\begin{array}{ccc} \pi & \rho & \left(N_{\text{bc}}\left(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}\right) + \phi_c\right) \\ +\pi & (1-\rho) & N_{\text{bc}}\left(\phi_c, \phi_p, 0, 1+\Delta_{\Phi_p}\right) \\ +(1-\pi) & \rho & N_{\text{bc}}\left(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}\right) \\ +(1-\pi) & (1-\rho) & N_{\text{bc}}\left(\phi_c, \phi_p, 0, 1-\Delta_{\Phi_p}\right) \end{array} \right). \end{aligned} \tag{50}$$

By definition, $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$ is the value of ϕ_c such that

$$N_{\text{bc}}\left(\tilde{\phi}^*_{c,\text{bc}}, \phi_p, 0, 1+\Delta_{\Phi_p}\right) = \frac{\mu}{1-\beta}. \tag{51}$$

[47] To economize on notation we typically write $\tilde{\phi}_{c,\text{no-bc}}$ instead of $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$.

[48] Assumption 2 ensures that the value of μ is low enough to ensure that entry is profitable as long as the efficiency condition is satisfied and the entry costs are low enough.

In a boom this type of job could either produce or not produce. The NPV would be equal to $\mu/(1-\beta)$ for both choices. In a recession this job cannot produce, so the revenues are equal to μ until the economy gets out of a recession at which point the NPV by definition is equal to $\mu/(1-\beta)$. Consequently,

$$N_{\text{bc}}\left(\tilde{\phi}^*_{c,\text{bc}}, \phi_p, 0, 1-\Delta_{\Phi_p}\right) = \frac{\mu}{1-\beta}. \tag{52}$$

Combining the last equations gives

$$\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right) = \frac{\phi_p\left(1+\Delta_{\Phi_p}\right)-\mu}{1-\beta\rho\pi}. \tag{53}$$

Assumption 2 directly implies that

$$\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right) < \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right) \text{ if } \tilde{\phi}_{p,\text{no-bc}} \leq \phi_p < \tilde{\phi}_{p,\text{bc}}\left(\Phi_-\right). \tag{54}$$

Proof of part 2 of proposition 3. To calculate the formulas in this part of the proposition, we have to first calculate the relevant NPVs. First consider a world without business cycles. If the job cannot satisfy the effort constraint or if the job creation costs are too high, then the job is never created and the revenues are μ each period. Thus

$$N_{\text{no-bc}}\left(\phi_c, \phi_p, 0\right) = \frac{\mu}{1-\beta} \text{ if } \phi_p < \tilde{\phi}_{p,\text{no-bc}} \text{ or } \phi_c > \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right). \tag{55}$$

If assumption 2 is satisfied, then job creation is profitable if the effort constraint is satisfied and the job creation costs are low enough. From equations (47) and (48), it follows that

$$N_{\text{no-bc}}\left(\phi_c, \phi_p, 0\right) = \frac{\phi_p - (1-\beta\rho)\,\phi_c}{1-\beta} \text{ if } \phi_p \geq \tilde{\phi}_{p,\text{no-bc}} = \chi \text{ and } \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right). \tag{56}$$

Now consider the case with business cycles. Here we focus on fragile jobs. If the job can never satisfy the effort constraint or if the job creation costs are too high, then the job will not be created and revenues are equal to μ. Thus,

$$\begin{aligned} \text{if } \phi_p &< \tilde{\phi}_{p,\text{bc}}\left(\Phi_+\right) \text{ or } \phi_c > \tilde{\phi}_{c,\text{bc}}\left(\phi_p, \Phi_+\right), \text{ then} \\ \mathrm{E}\left[N_{\text{bc}}(\phi_c, \phi_p, 0, \Phi_{p,t})\right] &= \frac{\mu}{1-\beta}. \end{aligned} \tag{57}$$

Now suppose that $\phi_p \geq \tilde{\phi}_{p,\text{bc}}\left(\Phi_+\right)$, $\phi_c \leq \tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)$, and (since we focus on fragile jobs) $\phi_p < \tilde{\phi}_{p,\text{bc}}\left(\Phi_-\right)$. The value for

$$\mathrm{E}\left[N_{\mathrm{bc}}(\phi_c,\phi_p,0,\Phi_{p,t})\right] = \frac{\left(N_{\mathrm{bc}}(\phi_c,\phi_p,0,1+\Delta_{\Phi_p}) + N_{\mathrm{bc}}(\phi_c,\phi_p,0,1-\Delta_{\Phi_p})\right)}{2}$$

is equal to

$$\mathrm{E}\left[N_{\mathrm{bc}}(\phi_c,\phi_p,0,\Phi_{p,t})\right] = \frac{1}{2}\left(\frac{\phi_p\left(1+\Delta_{\Phi_p}\right) + \mu - (1-\beta\rho\pi)\,\phi_c}{1-\beta}\right). \tag{58}$$

This formula follows from equation (50) and

$$N_{\mathrm{bc}}\left(\phi_c,\phi_p,0,1-\Delta_{\Phi_p}\right) = \mu + \beta\left(\begin{array}{cc} \pi & N_{\mathrm{bc}}\left(\phi_c,\phi_p,0,1-\Delta_{\Phi_p}\right) \\ +\,(1-\pi) & N_{\mathrm{bc}}\left(\phi_c,\phi_p,0,1+\Delta_{\Phi_p}\right)\end{array}\right). \tag{59}$$

The formulas in the proposition follow directly from combining the formulas for the appropriate NPVs.

For example, consider the case with $\tilde{\phi}_{p,\mathrm{bc}}\left(\Phi_+\right) \leq \phi_p < \tilde{\phi}_{p,\mathrm{bc}}\left(\Phi_-\right)$ and $\phi_c \leq \tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)$. We get that

$$\begin{aligned} L(\phi_c,\phi_p,\Delta_{\Phi_p}) &= \frac{(1-\beta)}{\phi_p}\left(N_{\text{no-bc}}(\phi_c,\phi_p,0) - \mathrm{E}\left[N_{\mathrm{bc}}(\phi_c,\phi_p,0,\Phi_{p,t})\right]\right) \qquad (60) \\ &= 1-(1-\beta\rho)\frac{\phi_c}{\phi_p} - \frac{1}{2}\left(\left(1+\Delta_{\Phi_p}\right) + \hat{\mu} - (1-\beta\rho\pi)\frac{\phi_c}{\phi_p}\right) \\ &= \frac{1-\Delta_{\Phi_p}-\hat{\mu}}{2} + \left(\frac{(1-\beta\rho\pi) - 2\,(1-\beta\rho)}{2}\right)\frac{\phi_c}{\phi_p}. \end{aligned}$$

First, consider the case when

$$\frac{(1-\beta\rho\pi) - 2\,(1-\beta\rho)}{2} \geq 0. \tag{61}$$

Then it follows directly from the third part of assumption 2 that $L(\phi_c,\phi_p,\Delta_{\Phi_p}) > 0$. Now consider the case when

$$\frac{(1-\beta\rho\pi) - 2\,(1-\beta\rho)}{2} < 0. \tag{62}$$

Since $\phi_c \leq \tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right) = \left(\phi_p\left(1+\Delta_{\Phi_p}\right) - \mu\right)/\left(1-\beta\rho\pi\right)$, we get that

$$\begin{aligned} L(\phi_c,\phi_p,\Delta_{\Phi_p}) &\geq \frac{1-\Delta_{\Phi_p}-\hat{\mu}}{2} + \left(\frac{(1-\beta\rho\pi) - 2\,(1-\beta\rho)}{2}\right)\frac{\left(1+\Delta_{\Phi_p}-\hat{\mu}\right)}{(1-\beta\rho\pi)} \qquad (63) \\ &= \frac{1-\Delta_{\Phi_p}-\hat{\mu}}{2} + \frac{\left(1+\Delta_{\Phi_p}-\hat{\mu}\right)}{2} - \frac{(1-\beta\rho)\left(1+\Delta_{\Phi_p}-\hat{\mu}\right)}{(1-\beta\rho\pi)} \\ &= 1-\hat{\mu} - \frac{(1-\beta\rho)\left(1+\Delta_{\Phi_p}-\hat{\mu}\right)}{(1-\beta\rho\pi)}. \end{aligned}$$

If the second part of assumption 2 is divided by χ, then it immediately follows that $L(\phi_c,\phi_p,\Delta_{\Phi_p}) \geq 0$.

C Implementation of empirical approach: details

C.1 Some definitions

- Mass of cyclical fragile jobs operating in world with business cycles:

$$E_{\text{C-fragile}} = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} f(\phi_c, \phi_p) d\phi_c d\phi_p.$$

  Associated output level evaluated at $\Phi_{p,t} = 1$:

$$Y_{\text{C-fragile}} = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p.$$

- Mass of permanent-loss fragile jobs:

$$E_{\text{PL-fragile}} = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_p)} f(\phi_c, \phi_p) d\phi_c d\phi_p.$$

  Associated output level evaluated at $\Phi_{p,t} = 1$:

$$Y_{\text{PL-fragile}} = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_p)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p.$$

- Mass of timed-entry jobs:

$$E_{\text{timed-entry}} = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty} \int_{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi -)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+)} f(\phi_c, \phi_p) d\phi_c d\phi_p.$$

  Associated output level evaluated at $\Phi_{p,t} = 1$:

$$Y_{\text{timed-entry}} = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty} \int_{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi.$$

- Mass of jobs not affected by timed-entry or the agency problem:

$$E = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)} f(\phi_c, \phi_p) d\phi_c d\phi.$$

  Associated output level evaluated at $\Phi_{p,t} = 1$:

$$Y = \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi.$$

- Employment level in a boom:

$$E_{\text{boom}} = E_{\text{C-fragile}} + E_{\text{timed-entry}} + E.$$

Associated output level evaluated at $\Phi_{p,t} = 1$:

$$Y_{\text{boom}} = Y_{\text{C-fragile}} + Y_{\text{timed-entry}} + Y.$$

C.2 Calculating the cost of business cycles

In this section, we calculate the cost of business cycles for timed-entry jobs, cyclical fragile jobs, and timed-entry jobs.

C.2.1 Timed-entry jobs

We calculate the cost of business cycles for two choices of the upper bound of the distribution of ϕ_c, $\overline{\phi}_c\left(\phi_p\right)$. The low value for $\overline{\phi}_c\left(\phi_p\right)$ is equal to $\frac{1}{2}\left(\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)+\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right)\right)$ and high value for the upper bound is equal to $\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)$ or any higher. At the low value for the upper bound implied by our choice of structural parameters, there are no timed-entry jobs and the effect of business cycles on timed-entry jobs would, thus, be zero. In the remainder of this section, we focus on the case when $\overline{\phi}_c\left(\phi_p\right) \geq \tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)$.

When considering timed-entry jobs, we have to distinguish between jobs with a value of ϕ_c above and below $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$. First, we calculate the output associated with the two types of timed-entry jobs.

$$\begin{aligned}
\frac{Y_{\text{timed-entry}}^{\phi_c>\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}}{Y} &= \frac{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\phi_c>\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)}\phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p}{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\underline{\phi}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)}\phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p} \\
&= \frac{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\phi_p\left(\int_{\phi_c>\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)} f(\phi_c|\phi_p)d\phi_c\right)f\left(\phi_p\right)d\phi_p}{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\phi_p\left(\int_{\underline{\phi}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)} f(\phi_c|\phi_p)d\phi_c\right)f\left(\phi_p\right)d\phi_p} \\
&= \frac{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\phi_p\left(\frac{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)-\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}{\overline{\phi}_c\left(\phi_p\right)-\underline{\phi}_c\left(\phi_p\right)}\right)f\left(\phi_p\right)d\phi_p}{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\phi_p\left(\frac{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)-\underline{\phi}_c\left(\phi_p\right)}{\overline{\phi}_c\left(\phi_p\right)-\underline{\phi}_c\left(\phi_p\right)}\right)f\left(\phi_p\right)d\phi_p} \\
&= \left(\frac{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)-\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)-\underline{\phi}_c\left(\phi_p\right)}\right)\frac{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\phi_p f\left(\phi_p\right)d\phi_p}{\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\phi_p f\left(\phi_p\right)d\phi_p} \\
&= \frac{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)-\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)-\underline{\phi}\left(\phi_p\right)}
\end{aligned}$$

The fractions in brackets do not depend on ϕ_p since each term depends linearly on ϕ_p. Consequently, they can be taken out of the integration.

The output measure for the other group of timed-entry jobs is given by

$$\frac{Y_{\text{timed-entry}}^{\phi_c<\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}}{Y}=\frac{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)-\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)}{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right)-\underline{\phi}\left(\phi_p\right)}.$$

Consider the timed-entry jobs that have a value of ϕ_c *above* $\tilde{\phi}_{c,\text{no-bc}}$. The welfare consequences are given by

$$\begin{aligned}
&\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)}\frac{L(\phi_c,\phi_p,\Delta_{\Phi_p})\phi_p}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p\\
=&\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)}\frac{L(\phi_c,1,\Delta_{\Phi_p})\phi_p}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p\\
=&\frac{X}{Y}\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}^{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)}\phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p\\
=&X\frac{Y_{\text{timed-entry}}^{\phi_c>\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}}{Y},
\end{aligned}$$

where

$$X=\mathrm{E}\left[\frac{N_{\text{bc}}\left(\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right),1,0,\Phi_{p,t}\right)-N_{\text{bc}}\left(\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right),1,0,\Phi_{p,t}\right)}{2}-\frac{\hat{\mu}}{1-\beta}\right].$$

For timed-entry jobs, the problem is linear in ϕ_p. Consequently, $L\left(\cdot\right)$, which is defined relative to ϕ_p, does not depend on ϕ_p, which implies $L(\phi_c,\phi_p,\Delta_{\Phi_p})=L(\phi_c,1,\Delta_{\Phi_p})$ and can be taken out of the integral. The NPV values in the expression for X are solved from equations 33 through 36, which is a linear system. X measures the average cost of business cycles for jobs with a value of ϕ_c in between $\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$ and $\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_+\right)$. This average cost is simply the average of the cost at the end points of the interval, since $L(\phi_c,\phi_p,\Delta_{\Phi_p})$ is linear in ϕ_c and the distribution of ϕ_c conditional on ϕ_p is uniform. The calculated cost is a gain, that is, $X<0$, independent of the value for ω_e.

The second type of timed-entry jobs are the ones that have a value of ϕ_c below

$\tilde{\phi}_{c,\text{no-bc}}$. There welfare consequences are given by

$$\begin{aligned}
&\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi-\right)}^{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}\frac{L(\phi_c,\phi_p,\Delta_{\Phi_p})\phi_p}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p\\
=&\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi-\right)}^{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}\frac{L(\phi_c,1,\Delta_{\Phi_p})\phi_p}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p\\
=&\frac{X}{Y}\int_{\tilde{\phi}_p(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi-\right)}^{\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}\phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p\\
=&X\frac{Y_{\text{timed-entry}}^{\phi_c<\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)}}{Y}
\end{aligned}$$

where

$$X=\mathrm{E}\left[\begin{array}{c}\frac{N_{\text{bc}}\left(\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right),1,0,\Phi_{p,t}\right)-N_{\text{bc}}\left(\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right),1,0,\Phi_{p,t}\right)}{2}\\ \\ -\frac{N_{\text{no-bc}}\left(\tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right),1,0,\Phi_{p,t}\right)-N_{\text{no-bc}}\left(\tilde{\phi}_{c,\text{bc}}\left(\phi_p,\Phi_-\right),1,0,\Phi_{p,t}\right)}{2}\end{array}\right]$$

This is a gain when $\omega_e = 1$, but can be a loss when $\omega_e < 1$. Entrepreneurs always benefit from the option to delay job creation, but workers may not if job creation is not efficient. Consequently, workers may suffer if business cycles induce these entrepreneurs to postpone job creation some of the times when they would never do so in a world without business cycles.

C.2.2 Cyclical fragile jobs

We first calculate the associated output measure, which is then used to calculate the cost of business cycles. Calculating the output that cyclical jobs produce is done in two steps. First, we calculate the *mass* of cyclical fragile jobs relative to total employment in a boom. Next, we calculate the associated output level.

From the data, we obtain a measure for Ξ_1, where Ξ_1 is defined as

$$\begin{array}{c}\text{mass of cyclical fragile jobs}\\ +\text{ mass of timed-entry jobs } \textit{exiting during downturn}\end{array}=\Xi_1 * E_{\text{boom}} \tag{64}$$

The relevance of timed-entry jobs depends on the choice for $\overline{\phi}_c\left(\phi_p\right)$. If the lower value for $\overline{\phi}_c\left(\phi_p\right)$ is chosen, then there are no timed-entry jobs for the calibrations considered here and we immediately get that

$$\frac{E_{\text{C-fragile}}}{E_{\text{boom}}}=\Xi_1. \tag{65}$$

Next, consider the case for which the upper bound of the distribution of ϕ_c is at (or above) the cut-off level for timed-entry jobs in a boom. This means that all potential timed-entry jobs are in the domain of ϕ_c. Using that the expected duration of a recession is eight quarters, we get that

$$E_{\text{C-fragile}} + XE_{\text{timed-entry}} = \Xi_1 * E_{\text{boom}}, \tag{66}$$

where

$$X = \left(\rho + (1-\rho)\rho + \cdots (1-\rho)^7 \rho\right). \tag{67}$$

We now can derive a lower bound for $E_{\text{C-fragile}}/E_{\text{boom}}$.

$$\begin{aligned}
&\frac{E_{\text{C-fragile}}}{E_{\text{boom}}} \\
&= \Xi_1 - \frac{X\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+)} f(\phi_c,\phi_p)d\phi_c d\phi_p}{E_{\text{boom}}} \\
&> \Xi_1 - \frac{X\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty}\int_{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+)} f(\phi_c,\phi_p)d\phi_c d\phi_p}{\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty}\int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+)} f(\phi_c,\phi_p)d\phi_c d\phi_p} \\
&= \Xi_1 - X\frac{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+) - \tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)}{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+) - \underline{\phi}_c(\phi_p)}\frac{\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty} f(\phi_c,\phi_p)d\phi_c d\phi_p}{\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}^{\infty} f(\phi_c,\phi_p)d\phi_c d\phi_p} \\
&= \Xi_1 - X\frac{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+) - \tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)}{\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+) - \underline{\phi}_c(\phi_p)}
\end{aligned} \tag{68}$$

The equality is replaced by an inequality, because we replace total employment in a boom, $E(\Phi_+)$, by the sum of regular and timed-entry employment, which is smaller.[49] To understand the last two steps, note that $\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_+)$, $\tilde{\phi}_{c,\text{bc}}(\phi_p,\Phi_-)$, and $\underline{\phi}_c(\phi_p)$ are linear in ϕ_p.

Now that we have calculated the *mass* of cyclical fragile jobs, we turn to the question how to calculate the *output* produced by cyclical fragile jobs. In our numerical work, we take a stand on Ξ_2, which is the productivity of cyclical jobs relative to the productivity of jobs not affected by the agency problems. That is,

$$\frac{\left(1+\Delta_{\Phi_p}\right)Y_{\text{C-fragile}}}{E_{\text{C-fragile}}} = \Xi_2\frac{\left(1+\Delta_{\Phi_p}\right)(Y_{\text{boom}} - Y_{\text{C-fragile}})}{E(\Phi_+) - E_{\text{C-fragile}}}.$$

From this we get

$$\frac{Y_{\text{C-fragile}}}{Y_{\text{boom}}} = \frac{\Xi_2\frac{E_{\text{C-fragile}}}{E_{\text{boom}}}}{1 - \frac{E_{\text{C-fragile}}}{E_{\text{boom}}}(1-\Xi_2)}.$$

[49] Since the number of cyclical jobs is bounded by the total observed variation in the number of workers over the cycle, the ratio $(E + E_{\text{timed-entry}})/E_{\text{boom}}$ cannot be very small so that the inequality introduces little slack.

Now that we have calculated the output measure, we calculate the cost of business cycles. The net loss for fragile jobs with $\phi_c \leq \tilde{\phi}^*_{c,\text{bc}}\left(\phi_+\right)$ is given by

$$\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{L(\phi_c,\phi_p,\Delta_{\Phi_p})\phi_p}{Y} f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{\left(1-\Delta_{\Phi_p}\right)\phi_p - \mu + \phi_c\left(1-\beta\rho\pi - 2\left(1-\beta\rho\right)\right)}{2Y} f(\phi_c,\phi_p)d\phi_c d\phi_p$$

$$+ \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{no-bc}}} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{-\left(1+\Delta_{\Phi_p}\right)\phi_p + \mu + \phi_c\left(1-\beta\rho\pi\right)}{2Y} f(\phi_c,\phi_p)d\phi_c d\phi_p \geq$$

$$\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{-\Delta_{\Phi_p}\phi_p + \phi_c\beta\rho\left(1-\pi\right)}{2Y} f(\phi_c,\phi_p)d\phi_c d\phi_p = \tag{69}$$

$$\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{-\Delta_{\Phi_p}\frac{+\left(\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)-\underline{\phi}_c(\phi_p)\right)\beta\rho(1-\pi)}{2\phi_p}}{2Y} \phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$\int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{-2\Delta_{\Phi_p} + \left(\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right) - \underline{\phi}_c\left(\phi_p\right)\right)\beta\rho\left(1-\pi\right)}{4\phi_p Y} \phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$\frac{-2\Delta_{\Phi_p} + \left(\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right) - \underline{\phi}_c\left(\phi_p\right)\right)\beta\rho\left(1-\pi\right)}{4\phi_p} \int_{\tilde{\phi}_{p,\text{bc}}(\Phi_+)}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c(\phi_p)}^{\tilde{\phi}^*_{c,\text{bc}}(\phi_p,\Phi_+)} \frac{\phi_p}{Y} f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$\frac{-2\Delta_{\Phi_p} + \left(\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right) - \underline{\phi}_c\left(\phi_p\right)\right)\beta\rho\left(1-\pi\right)}{4\phi_p} \frac{Y_{\text{C-fragile}}}{Y} \geq$$

$$\frac{-2\Delta_{\Phi_p} + \left(\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right) - \underline{\phi}_c\left(\phi_p\right)\right)\beta\rho\left(1-\pi\right)}{4\phi_p} \frac{Y_{\text{C-fragile}}}{Y_{\text{boom}}}$$

In the first row, the value of $L\left(\cdot\right)$ is multiplied by ϕ_p since $L\left(\cdot\right)$ is scaled by ϕ_p. The expressions used for $L\left(\cdot\right)$ are given in proposition 3. The first inequality is based on the assumption that the mass of jobs below $\tilde{\phi}_{p,\text{no-bc}}$ does not exceed the mass above $\tilde{\phi}_{p,\text{no-bc}}$ and on using the welfare loss (gain) for jobs at $\phi_p = \tilde{\phi}_{p,\text{no-bc}}$ which are lower (higher) than the welfare losses (gains) for other fragile jobs.[50] The latter reduces little slack in the calculations at least for the parameter values we explored. The reason that $\left(\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right) - \underline{\phi}_c\left(\phi_p\right)\right)/\phi_p$ can be taken outside the integration is that the numerator is linear in ϕ_p so that the ratio does not depend on ϕ_p.

[50] See equation (30).

C.2.3 Permanent-loss fragile jobs

First, we calculate the output that could be generated by fragile jobs with a value of ϕ_c such that $\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}\left(\phi_p\right)$. If a job's value of ϕ_c is above $\tilde{\phi}_{c,\text{bc}}$, then it is too high to make entry profitable in a world with business cycles. Consequently, we do not observe these jobs in the real world. These jobs, however, have one characteristic in common with the fragile jobs we do observe and that is their productivity level, ϕ_p. In particular, the productivity levels of these jobs are in between $\tilde{\phi}_{p,\text{no-bc}}$ and $\tilde{\phi}_p(\Phi_-)$, which means that they are in the upper half of the productivity levels of the fragile jobs that produce $Y_{\text{PL-fragile}}$. Given our assumptions on $f(\phi_c|\phi_p)$, it is then straightforward to calculate $Y_{\text{PL-fragile}}$.

$$\begin{aligned} & \frac{Y_{\text{PL-fragile}}}{Y} \\ = & \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right)}^{\overline{\phi}_c\left(\phi_p\right)} \frac{\phi_p}{Y} f(\phi_c, \phi_p) d\phi_c d\phi_p \\ = & \frac{X}{Y} \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_-)} \int_{\underline{\phi}_c\left(\phi_p\right)}^{\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p,\Phi_+\right)} \phi_p f(\phi_c, \phi_p) d\phi_c d\phi_p \\ \geq & \frac{X}{2}\frac{Y_{\text{C-fragile}}}{Y} \geq \frac{X}{2}\frac{Y_{\text{C-fragile}}}{Y_{\text{boom}}}, \end{aligned} \tag{70}$$

where

$$X = \frac{\overline{\phi}_c\left(\phi_p\right) - \tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right)}{\tilde{\phi}^*_{c,\text{bc}}\left(\phi_p, \Phi_+\right) - \underline{\phi}_c\left(\phi_p\right)}.$$

X does not depend on ϕ_p so can be taken out of the integral. The inequality follows directly from assumption 3, which states that the mass of fragile jobs with a value of ϕ_p below $\tilde{\phi}_{p,\text{no-bc}}$ does not exceed the mass of fragile jobs with a value of ϕ_p above $\tilde{\phi}_{p,\text{no-bc}}$.

The welfare loss of fragile jobs with a value of ϕ_c above $\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)$ is given by

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_p(\Phi_-)}\int_{\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)}^{\overline{\phi}_c\left(\phi_p\right)}\frac{L(\phi_c,\phi_p,\Delta_{\Phi_p})\phi_p}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p = \tag{71}$$

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_p(\Phi_-)}\int_{\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)}^{\overline{\phi}_c\left(\phi_p\right)}\frac{\phi_p-\mu-(1-\beta\rho)\,\phi_c}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_p(\Phi_-)}\int_{\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)}^{\overline{\phi}_c\left(\phi_p\right)}\frac{\phi_p-\mu-(1-\beta\rho)\,\frac{\tilde{\phi}_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)+\overline{\phi}_c\left(\phi_p\right)}{2}}{Y}f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_p(\Phi_-)}\int_{\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)}^{\overline{\phi}_c\left(\phi_p\right)}X\phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$X\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_p(\Phi_-)}\int_{\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)}^{\overline{\phi}_c\left(\phi_p\right)}\phi_p f(\phi_c,\phi_p)d\phi_c d\phi_p =$$

$$X\frac{Y_{\text{PL-fragile}}}{Y},$$

where

$$X=\frac{1-\hat{\mu}-(1-\beta\rho)\,\frac{\tilde{\phi}^*_{c,\mathrm{bc}}\left(\phi_p,\Phi_+\right)+\overline{\phi}_c\left(\phi_p\right)}{2\phi_p}}{Y},$$

which does not depend on ϕ_p.

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